Organodynamics: A General Theory of Dynamical Systems based on Chance Organization

Part I of VI: Overview

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Grant Holland; Santa Fe Alliance for Science; Santa Fe, New Mexico;
email: grant.holland at organiccomplexsystems.org; April, 2014.

To Predictively Model Massively Complex Systems

Imagine trying to predict the unfolding behavior of the following complex systems over time: the evolution of a moth species recently identified in the upper Amazon basin; the development of an incipient tropical system off the west coast of North Africa on a warm day in September; how a cloud computing data center must change its configuration over time in order to accommodate evolving demand and constraints on resources; or the patterns of neural activity that develop in a human subject while working a crossword puzzle, as captured by functional magnetic resonance imaging.

Predicting the evolution of the patterns of organization resulting in any of these systems is beyond the reach of our most contemporary systems theories. Our mature classical theories such as Hamiltonian mechanics can make consistent and precise predictions concerning well-behaved linear systems. And even certain surprisingly complex systems can be modeled with some fidelity using nonlinear dynamics and chaos theory, as long as the behaviors are deterministic.

But the systems just mentioned are not so well behaved, even while being very pertinent to our cultural, business, engineering or scientific interests. Why can't our systems theories do a better job of predicting how these types of complex systems will unfold?

Are our current theories perhaps trapped in mathematical traditions and habits that emphasize numerical precision of minutiae, while the prediction of more flexible, adaptable and proximate systemic behavior would be more practical and useful? Are these theories focused too much on trajectories of point particles, rather than on a concept of whole-system state change under nondeterministic conditions?

This paper presents a new approach to the modeling of complex dynamical systems. It focuses on defining *state* as whole-system organization, rather than as a property of individual point particle constituents. And, it articulates system dynamics in a more flexible, permissive and inclusive manner, even while focusing on the constraints imposed on their behavior - and attending to mathematical rigor.

The theory presented is a work in progress and leaves a number of issues unresolved. But it suggests a new avenue toward the investigation and modeling of massively complex dynamical systems, and hopefully suggests some promising direction.

Introducing Organodynamics

This article is the first of a series of five that introduces and describes a new dynamical system theory named *organodynamics*.

We shall begin by motivating the need for this new theory, and then describe the theory's particular perspective on what constitutes a dynamical system. We shall explain which particular aspects of dynamical systems *organodynamics* finds salient and chooses to emphasize. As with any dynamical systems theory (DST), its choice of which features to emphasize differentiates organodynamics from other DSTs and determine which domains of application it best fits. Also, we shall present a strategy for developing a mathematical formalism for the ideas presented.

Motivation for the New Theory

In the first decade of this century, the author worked as a systems engineer for a corporation whose principle product was server computers and "server farms", and whose typical customers were other corporations that purchased scores, hundreds or thousands of those servers to populate their data centers. Like the furniture company that sells "rooms to go" (whole rooms at a time), it had emerged that my employer had found itself in the business of designing, manufacturing and selling "data centers to go".

Data Center Management

The IT departments of these corporations had the task, on a continuing basis, of accepting requests from various user organizations for some number of server computers to be used for a specific project for a specified period of time (days, weeks, months, etc.) Of course, the total compendium of servers in the data center would be partitioned, for some time period, into clusters of these servers – each cluster being temporarily assigned to each of these concurrently using organizations. Such a partitioning of the data center would represent a high-level *configuration*, or *organization*, of the data center.

Beneath that highest level of organization, each cluster needed to be configured with a set of servers for its user team so that each of the servers played a particular role, as define by that user team. And, within each cluster, the individual server computers also required their own configuration, or organization.

In other words, at any moment in time, the data center had an *organization*, or *configuration*. And it was a complex organization, being comprised of organization within organization within organization of hardware or software entities, to some finite number of levels of organization.

In fact, a precise definition of what this organization was at any point in time – its *configuration* or *constellation of relationships*, is the *overriding concern* of the management of that data center – and of the IT department of any corporation that became a customer of my employer.

It is important to take note that this *constellation of relationships*, this *organization* of the data center, is NOT reducible to being described by a simple number, or even a vector of numbers – at least without loosing the information of interest, which are the interrelationships of its elements. Rather, such organizations could only be described as a constellation (or possibly, network) of interrelationships.

In other words, defining the *state* of such a system by using a single number – or even a vector of numbers – does not work for this application. Such a definition of *state* does not describe the data center in any way that is adequate to address the problem at hand. What is needed instead is a definition of state that captures the current constellation of interrelationships among the system's components.

But the IT department has a more interesting problem even than this. It is primarily interested in how the configuration of the data center, and all of its nested sub-organizations, *changes over time*. In fact, you would have to say that the primary task of these IT departments – data center management – is precisely the task of managing the change of organization of the data center over time.

Of course, this problem has an addition complication – *uncertainty*. It is not certain how the organization of the data center will need to change over time, because it is not certain what requests for computer use by various user teams will occur over time. Moreover, there are also various aspects of the data center itself - which also, are not predictable. These include, for example, the reliability and availability of the configurable equipment.

Not surprisingly, my employer was not permitted by these corporate customers to simply remain in the business of "designing, manufacturing and selling server computers". Rather, these corporate customers demanded serious help from my employer for the process of managing their data centers - managing these dynamically changing configurations and reconfigurations, or *organizations* and *reorganizations*.

Thus, it fell to my group of systems engineers to design technologies, products and methodologies that enable our customers to manage their data centers. Our first step was to understand the problem space by constructing a system model. After some reflection, it became evident that the principle entity of interest within this problem space is that of the *configuration*, or *organization*, of the data center and its entities.

In fact, what we had here is some kind of dynamical systems whose principle entity of interest is an *configuration of entities*, and whose *state* a any moment in time must be characterized by some description of *how that configuration is organized*. Specifically, such a description of *organization* should include the interrelationships among the entities involved in the configuration. What is noteworthy is that such a *configuration*, as an entity, is not a point-like "particle", but is rather a rich constellation of relationships in its own right.

Biological Systems

Clearly, then a model of "configuration management" of a large data center is some kind of complex dynamical system whose principle object of interest is an entity that describes how the data center is organized (configured) at some point in time. Such an entity – which we are calling an *organization* - is complex in its own right. Rather than being a simple point-like particle, an *organization* is a complex constellation of interrelationships that cannot be reduced to a single number, or even to a single vector.

The question arises "Is data center configuration management" an isolated example of such a complex dynamical system; or are there others?" One needs look no further than the realm of biological systems to find another.

It seems that a reasonable viewpoint regarding biological systems is that one of their more salient features is the ways that they are organized, and the ways that they change those organizations over time. Moreover, this is the primary interest at every

level of abstraction within biological systems – from the macromolecular to the ecosystemic.

A second salient feature that biological systems have in common with data center configuration management is that of the unpredictability of how it will be reorganized in the future. This is true at the simplest prokaryotic cellular level as well as with the most complex multicellular organisms, as well as within and across ecosystems.

At this point, then, we have two exemplar systems that share some commonality: Their most conspicuous, compelling and salient qualities are 1) they are primarily characterized by the way that they are organized, and 2) their organizations change over time, and 3) the ways that their organization changes is often subject to chance.

Since we shall often be referring to these three systems theory qualities, it will be convenient to name them. Respectively, we shall I call them 1) *organization*, 2) *reorganization*, and 3) *chance reorganization*. For brevity, we shall also name the group of all three as *chance organization*. (The reader will note that this last phrase appears in the subtitle of this article.)

Search for an Apropos Dynamical Systems Theory

The development of products and technologies to automate data center configuration management would be well served by utilizing a dynamical systems theory as an intellectual foundation. Clearly, at the very least, such a dynamical systems theory would need to feature the three properties we just named, which we collectively called *chance organization*.

Below, we shall survey a few established theories of dynamical systems to see if they provide support for *chance organization*, and its three properties of *organization*, *reorganization* and *chance reorganization*.

The reader will not be surprised that our survey will not be successful in finding an established dynamical systems theory that supports chance reorganization. And therefore it will be necessary to create and articulate a new dynamical systems theory that does. This new dynamical systems theory is named *organodynamics*; because of its emphasis on the idea of *organization* and *organizational change*.

A Perspective on the Elements of Dynamical Systems Theories

I have used the notion of a “dynamical system” above rather loosely to mean a system that has the notion of *state*, and that addresses how that state changes over time. Moreover, it has become expected that any *dynamical systems theory* put forth some mechanism – the *dynamics* of the system - by which such change-of-state is promoted. In addition, these notions of state, state change and dynamics, should be given a mathematical articulation by the theory in question.

An Adopted Interpretation of Dynamical Systems

This is a reasonable meaning, and one that I intend. However, I must also deal with the fact that the term “dynamical system” has evolved to take on a much narrower connotation in systems theory, applied mathematics and physics literature. This more narrow and widely adopted meaning serves to accommodate the available and powerful mathematical tools that have developed over the previous couple of centuries from the combined fields of mathematical analysis, topology and linear algebra.

These tools include differential and integral equations, manifolds, and normed linear spaces such as Banach and Hilbert spaces. A phrase that I have seen that pretty well characterizes this typical approach to dynamical systems is “deterministic linear time-invariant differential systems”.

However, this is not a description that characterizes the above data center management exemplar. Nor is it a description that characterizes the perspective of biological systems as described above.

To see this, consider the entity that is at the center of focus of the data center management example. That entity is *a configuration of computers*. We named this configuration an *organization*. Such an entity is a *complex constellation of interrelationships*. Its state is not well described by a number; nor is it well described by a vector of numbers. Yet, this *organization* is the fundamental entity of interest in the data center example.

Moreover, the *space of interest* in this system is the *set of all such organizations*. This is not a space of numbers; nor is it a space of vectors. It is not a manifold. Rather, the members of this space are all of the possible *organizations* of the data center. Each such member of this space is, then, a possible *configuration* of the data center.

Lets look at this state space of our data center management problem. It is constituted by the set of all possible configurations of the data center – including hardware and software. This is a very large, but finite space, whose members are each very complex. Notice that there is no obvious or natural ordering of these configurations. And there is also no obvious notion of “distance” between any two such configurations; so we don’t even have a metric space. This means that the space is not discernably measureable, in the sense of the Lebesgue theory. This implies that notions of continuity and of differentiability do not apply in any obvious way. Moreover, the requirement to support chance variation by our data center example obviates determinism (if by “determinism” is meant “no chance variation”).

So, it is safe to say that the more widely adopted notion of dynamical systems being “deterministic linear time-invariant differential systems” does not hold very well for our data center example, or for the viewpoint for biological systems that we described above.

Nevertheless, our data center management example does comply with the broader notion of *dynamical system* that I put forth above. And, it does exhibit very rich and complex mathematical structure – just not deterministic, linear, time-invariant, or differentiable structure. In fact, the role of organodynamics as a dynamical systems theory is to provision the kinds of structures exhibited by our above exemplars systems.

So, it behooves us to present a broader view of what is meant by *dynamical systems* in these articles – a view that will, of course, include deterministic linear time-invariant differential systems, but that will also include our data center management and biological systems viewpoint as well.

A Broad Interpretation of Dynamical Systems

By *dynamical systems theory* I shall include any mathematical systems theory that features the following ideas: system members (elements), element *interrelationships*, *state*, *state space*, and *trajectory*. The notion of trajectory defines a set of *states* that an element can assume over some time interval.

In addition, any dynamical systems theory must specify the dynamics of the systems that it models. By *dynamics* is meant a *mathematical mechanism* that describes how the state of the system changes, or evolves, over time. These mechanisms typically take the form of *equations of motion*. They take a set of initial conditions as input values that represent the state of the system at some particular point in time. The equations of motion then calculate the precise state of the system at all future points in time using the algorithm inherent in the equations of motion together with the initial conditions. In fact, it is the dynamics of a DST that specify its *trajectory*.

DSTs that provide such a precise time evolution are called *deterministic*. Classically, most DSTs have been deterministic in this regard. Examples of such deterministic dynamical systems theories are Newtonian and Hamiltonian mechanics, and nonlinear dynamics (otherwise known as *chaos theory*).

However, organodynamics takes certain liberties with the notion of dynamics. These liberties involve a relaxation, or generalization, of the concept to merely require that the dynamics of the system merely *constrain* the future outcome to some limited set – rather than insist that they be precisely determined. Of course, being *precisely determined* is still allowed as a special case – the *deterministic* case.

Many mathematical mechanisms can be employed to merely constrain these future outcomes. An example of such a mechanism is *inequalities*. Mathematical "inequalities of motion" could be use to constrain the time evolution of a dynamical system.

Another possible mathematical mechanism for providing non-deterministic *dynamics* is the *probability space* – abbreviated to its *probability distribution*. This mechanism, in fact, is the one employed by organodynamics. The probability space defines a sample space within which the possible outcomes of the time evolution of the dynamical system can be constrained. Additionally, the probability space provides a probability measure than can provide a kind of "weighted" constraint on the time evolution of a dynamical system.

In this way, the probability space can be used as a constraining mechanism to describe the time evolution of a system in a more general, and permissive, manner than a strictly deterministic mechanism. Additionally, the determinism can be shown to be a special case of the probability space – the case when all sample points have probability of zero, except for one sample point, which has probability of 1. This is the so-called *constant distribution*.

Thus, organodynamics shall take this more general understanding of *system dynamics*, which allows the dynamical mechanism of the theory to merely *constrain* the time evolution of the systems that it models – rather than requiring that it determines that time evolution precisely. Moreover, the mathematical mechanism that organodynamics uses to impose these constraints is the probability space – normally abbreviated to a probability distribution.

So we shall amend our meaning of *dynamical systems theory* to mean a mathematical systems theory that features the following ideas: system elements, element *interrelationships*, *state*, *state space*, and *trajectory* and *system dynamics*[1].

[1] Some physicists consider quantum mechanics to be "deterministic" because of the fact that the probability distribution of the state of the system can be determined. However, I consider a dynamical system to be deterministic only if the *state of the system* itself can be precisely determined through its *dynamics* (equations of motion). Being able to "determine" any other aspect of the dynamical system (such as its probability distribution) is not enough to qualify the theory as "deterministic". Obviously, there are many

The first task of a DST, as described, is to identity some aspect of the system-of-interest that shall be subject to change over time, and then to identify some mathematical construct that shall represent that entity. This mathematical construct is called the *state* of the entity. Secondly, another mathematical construct must be defined that represents how that state changes over time. It is this second construct that I shall refer to as the *trajectory* of the system. Change in state, or *trajectory*, is a central focus of any dynamical systems theory – at least according to the interpretation we are using.

A dynamical systems theory, then, represents the overall changes over a time period as the set of system states that are assumed over that time period – with one state in the set for each moment in time represented in the time period. The theory may choose to use either discrete time points or continuous time. Such a set of states is called the *trajectory* of the element over the time period. In other words, the *trajectory* models the *time-evolution* of the changes. It is fair to say that the characterization of the time-evolution of change is a primary goal of any dynamical systems theory. Thus, the notion of trajectory is essential to any DST.

The view that these articles take of dynamical system theories, then, posits these three central ideas – *state*, *trajectory and* dynamics – as the essential devices that a DST uses to represent change in a system-of-interest.

Another issue is "Which entity type of a dynamical system does a theory track the changes of." Typically, a DST will track how an individual element of the system changes over time. For example, in classical dynamics, the elements are *particles in Euclidean space* (actually, particles in a manifold); and that theory is interested in tracking their changes. As we shall see below, classical dynamics represents the *state* of a particle as a six-dimensional vector.

But, some theories assign *state* to the system-as-a-whole, and track its changes. Classical and Hamiltonian dynamics both do this by using a very large vector that has six dimensions for each of the particles in the space. So, both theories can either track the state of one particle at a time, or of an entire space. Organodynamics, however, assigns state only to a system. Its *state* defines "the way that the system is currently organized".

Lets summarize by saying that in the present series of articles we shall use the phrase *dynamical systems theory* to mean one that supports the concepts of state, trajectory and dynamics. Some DSTs focus on the states and trajectories of individual elements (particles), others on the states and trajectories of the system-as-a-whole, while others provide both views.

As mathematical theory, a DST must define the underlying space in which these elements reside. For example, the underlying space of classical mechanics is the set of all points (described as vectors of real numbers) in a manifold (e.g. Euclidean space or more generally a Hilbert space).

In our data center management example, the underlying space is the set of all possible *configurations* of the data center. Such a *configuration* is best defined by a much richer construct than a simple real number, or even a vector of them. A better candidate

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aspects of a dynamical system – besides its state – that can be determined. But if the dynamics (equations of motion) of the theory cannot precisely determine the future time evolution of the *state of the system*, then I regard labeling the theory as "deterministic" to be specious. By that consideration, then, I do not regard quantum mechanics as "deterministic".

would be a much richer mathematical entity – such as a network (or directed graph). Another reasonable candidate would be a topology on an underlying set[2].

So the concepts presented above are the basic interpretation of *dynamical systems* that is made by these articles, and is what is meant in this paper by this terminology. Admittedly, these ideations make for a more generous notion of DST's than does perhaps the general usage of the phrase. However, organodynamics needs to be flexible and generous in what it means by *dynamical systems*.

These concepts are the minimum and essential ones that we shall use to describe and compare some of the established and adopted theories of dynamical systems exemplified below. We shall use these basic concepts to introduce organodynamics in the present paper. In the later papers of this series, we shall elaborate considerably on these basic ideas, how organodynamics defines them, and how the specific treatment of these ideas in organodynamics enables the modeling of specific classes of complex systems, and how these factors differentiate organodynamics from other DST's.

It is understood that there are many reasonable perspectives on what constitutes a dynamical systems theory, some of which are not precisely congruent with the one presented here. However, the author believes that the viewpoint on dynamical systems theories presented here is a reasonable one. And that this characterization lends itself to modeling a particular class of complex dynamical systems – a class that is targeted by organodynamics.

In the next section, we shall review at a very high level some established and important theories of dynamical systems in order to characterize how each treats the essential ideas discussed in the present section. This discussion will set the background for defining the way in which organodynamics will treat these conceptualizations, and how and why it differentiates itself from these more traditional dynamical systems theories.

How Some Established Theories Portray these Elements.

In this section, we shall look at how three particular established theories articulate, in their own ways, the notions discussed above: system members or *elements*, element *interrelationships*, *state*, *state space*, *trajectory* and *dynamics*.

We shall look at a few exemplar theories, all of which qualify as dynamical systems theories according to the characterization of DST's taken by these articles and presented in the previous section. We shall briefly discuss three such theories: 1) classical and Hamiltonian mechanics, 2) statistical mechanics, and 3) nonlinear dynamics (or, chaos theory). Other important examples come to mind – including *quantum mechanics*, and also including *stochastic dynamical systems*. However, the three theories selected will be enough to preview the range of choices that can be made by a theory to represent system elements, element interrelationships, definitions of state, and trajectory within a dynamical system as we have defined these ideas above.

This paper sees the need to go through this inspection - because discerning the similar but distinct ways that several theories each handle the above-mentioned issues will

[2] As we shall show in a later article of this series, organodynamics shall invent and use a construct that includes both ideas at once – a kind of "extended topology". In other words, the concept of "system state" in organodynamics will be defined as a topology. And, the *state space* of organodynamics will be defined as the set of all possible such "extended topologies" on some underlying set.

assist in an understanding of how and why they each carves out its own peculiar domains of application, and why we need each of them.

This will set the stage for an understanding of the distinctive role that is to be played by organodynamics as an adjunct to the others. Not a replacement, or a competitor, but an adjunct. We shall see how each theory's choice of the way it treats these issues determines which domains of application it is best suited to.

Classical and Hamiltonian Mechanics

Classical mechanics models a system whose elements are "point-like" *particles*, each of which is attributed a *state* that is specified by the *location* of the particle in conjunction with its *velocity*. A single six-dimensional state vector in manifold models these particle attributes. This vector consists of three coordinates for position (the three dimensions of space) as well as three more coordinates for velocity along the same three dimensions of space. (Physicists often use the term "coordinates" for the first three, but not for the second three.)

The dynamics of classical mechanics describes how and where within this space these particles can simultaneously move around by continuously changing their positions and velocities. This initiates the notion of the *state space* of these particles, which represents a "snapshot" in time of the states of all of the particles.

This *state space* is a manifold, with a notion of distance defined on the elements. This metric is the principle relationship bestowed on the elements of classical mechanics. And, while the cardinality of this state space is that of the continuum, the cardinality of the set of particles themselves within this space is finite - usually very large, but finite [Penrose, Oliver 2005].

This notion of distance sets up the possibility that two of the particles could collide – or be theoretically co-located. However, this potentiality complicates the dynamics significantly and is not emphasized within the theory. These considerations are generally left for the discipline of thermodynamics. Collisions are a prominent consideration in statistical mechanics, which develops a considerable degree of mathematical machinery toward particle collisions. On the other hand, the set of particles considered are sometimes organized into subclasses according to some specific systemic property. For example, all particles of a specific energy level may be considered as a subset for a variety of reasons [Tolman 1938].

The concept of *trajectory* is emphasized by classical dynamics. A *trajectory of a particle over some time* period is the set of all 6-dimensional points (states) that the particle assumes over the period. This marks out a continuous function over the time segment in question. In fact, such a function is generally differentiable and integrable.

As well as the notions of *state* and *trajectory* of a particle (elements), classical mechanics extends these ideas to the system of particles as a whole. Creating an enlarged idea of *state* wherein all six dimensions of each particle in the space are concatenated into one large state vector does this. For example, if the system consists of 1 billion particles, then the *system state* is represented by a vector with 6 billion dimensions – one for each of the six dimensions of each particle.

In the 18th and 19th centuries, a reformulation of classical mechanics was put forth by Lagrange and later reinterpreted by Hamilton. These reformulations were more abstract and more general than the classical theory. The Hamiltonian version has largely replaced the classical, or Newtonian version in the recent history of mechanics

and dynamical systems. At the risk of skipping the significance of these reformulations, and in the interest of the subject at hand, we shall proceed to simply describe some of the differences in how the notions of state, trajectory, etc. are defined and treated in Hamiltonian dynamics as compared to the classical treatment.

We shall merely state here that the notion of the state vector of six dimensions is preserved in Hamiltonian dynamics, but three of these dimensions are redefined. The first three dimensions still represent the location, or position, of a particle in a 3-dimensional Euclidean space. However, the 4^{th}, 5^{th} and 6^{th} dimensions have been changed to represent *momentum*, rather than *velocity*. Of course, this fact effectively brings mass into the picture because momentum equals velocity times mass. To say more about this here would take us outside the scope of this paper. However, suffice it to say that the concept of *trajectory* in Hamiltonian mechanics is likewise adjusted, so that it is the path of a Hamiltonian "point" rather than a classical, or Newtonian, one.

Classical mechanics establishes the equations of motion that determine the dynamics of the particles in the system. Hamiltonian mechanics deals directly with the concept of energy as having a determination on the trajectory, or time evolution, of a particle. Given a particle and its initial state, Hamiltonian mechanics defines a calculation known as *the Hamiltonian* that is a function of the initial state and the time. It turns out that the Hamiltonian corresponds to the total energy of the particle. This value is then provided as the input to the equations of motion as defined in Hamiltonian mechanics. In this way, then the energy of the particle as calculated from the initial state of the particle determines the time evolution of the particle. In fact, this idea is also extended to calculate the trajectory of the whole system of particles.

Both of these approaches enable the respective theory to calculate precisely the trajectory of a particle, or its time evolution, having been given its state at the beginning of the trajectory's time interval. This fact leads us to categorize these two theories as *deterministic*. Because of these dynamics, it is fair to describe classical and Hamiltonian dynamics as emphasizing deterministic linear time-invariant differential systems.

Statistical Mechanics

In the last half of the 19^{th} century, the atomic theory of matter began to achieve broad adoption. This adoption imposed a difficulty on the physics of thermodynamics, because the atomic theory says that physical entities are composed of a finite but vast number of elements (atoms and molecules) – so many in fact that it impossible to say where any of them is at any given time. Of course this fact meant that the elements of mechanics, such as state and trajectory, could not be calculated by the relationships of classical or Hamiltonian mechanics, because the initial state of atoms could not be known precisely at any point in time. It's fair to say that statistical mechanics arose at that time to address those issues. Statistical mechanics addressed these issues by taking a statistical approach, rather than the deterministic one of classical and Hamiltonian mechanics.

In the interests of time and space, I shall describe only how statistical mechanics differs from classical and Hamiltonian mechanics in its treatment of the elements of dynamical systems as listed above. Of course, there are many reasonable interpretations of statistical mechanics, and its place in the world. As above, I am presenting one here that serves the interests of these articles by highlighting certain aspects - aspects that are be emphasized by organodynamics.

Admittedly, I am influenced by the interpretation of Richard Tolman [Tolman 1938], who postures statistical mechanics as a mathematical analog of classical thermodynamics – wherein thermodynamics loses its determinism and takes on a decidedly stochastic character.

Statistical mechanics does use Hamiltonian physics as the basis of it dynamics. This aspect is, of course, deterministic. However, what are statistical in the picture are the initial conditions of the particles in the space, and the uncertainties that reverberate therefrom – including the stochastic aspect passed on to particle collisions. Of course, this stochasticism is due to the limitations of the human mind – the scientist, the observer – to have knowledge of initial conditions of all of the particle in the state space, which is information necessary to feed into the equations of motion of Hamiltonian mechanics.

Nevertheless, considering both the dynamics and the initial conditions, the overall situation is nondeterministic – statistical or stochastic. That is, in statistical mechanics, there is uncertainty regarding the Hamiltonian states of the particles at any point in time. This was an early moment in the history of science wherein stochasticism found a foothold in a broadly adopted physics theory.

However, statistical mechanics *does* presume to know the probability distributions of the states of the particles. (It provides a theoretical development of them based on a theorem by Liouville.) And using these, statistical mechanics is able to make statements regarding the probable states of both the particles and of the particle space (the state space) as a whole. As well, statistical mechanics is able to deduce the probabilities of various trajectories of these particles and their state spaces. Of course, each of these trajectories is the member of a complex probability space – distinct from the space consisting of the particles themselves. Nevertheless, this derived probability space, whose members are trajectories, has its own probability distribution. And these probabilities, also, can be derived within the calculus of statistical mechanics.

Now, the elements of this calculus, as described, are *particles* and *trajectories* – which places this calculus within the physical realm of *mechanics*. However, from this basis, statistical mechanics contrives equivalence transformations that map the fundamentals of classical thermodynamics to correlated concepts within statistical mechanics. Specifically, in order for statistical mechanics to function as an analog to classical thermodynamics within a world explained by the atomic theory, five specific concepts (or *thermodynamic functionals*) from of thermodynamics must have counterpart correlates in statistical mechanics. These concepts are: energy, work, heat transfer, entropy and temperature [Tolman 1938].

A great accomplishment of statistical mechanics is that it has been able to develop the mathematics of these invariants, equivalence relationships or *analogs* between the world of classical thermodynamics and the new world of statistical mechanics for these five concepts. However, the classical world is one of *certainty* – especially regarding state, trajectory and these five thermodynamic functionals. On the other hand, the world of statistical mechanics is one of uncertainty – uncertainty about state, trajectory and its correlates of the five thermodynamic functionals.

Notice that a conspicuous and significant difference between classical and Hamiltonian mechanics and statistical mechanics it that statistical mechanics is *probabilistic* while the other two are deterministic. In fact, it is fair to describe statistical mechanics as a *stochastic* linear time-invariant differential system. In these papers, I shall use additional vocabulary to specify these differences. This vocabulary will include

alternatives to the term *probabilistic*, such as *statistical*, *random*, *non-deterministic* and *stochastic*. Otherwise, for theories such as for classical and Hamiltonian mechanics, I shall use the terms *deterministic* and *non-random*.

Nonlinear Dynamics

Nonlinear dynamics arose as a result of noticing that, within many complex dynamical systems, small differences in the way those processes begin can result in surprising differences after they have been running for awhile. Specifically, if the same such process is run twice with the same small differences in initial conditions, then the two results after the same amount of time is often large – larger than one would "expect". This phenomenon is called *sensitivity to initial conditions*.

However, intentionally, nonlinear dynamics confines its interests to processes that are *deterministic*. This means that, despite the fact that the process results in surprising behavior because it is "sensitive to initial conditions", its results nevertheless are always the same at every step of the way, even though it is given the *exact* same initial conditions each time. This kind of behavior is certainly not "chaotic" by any popular definition of the word.

Thus, nonlinear dynamics evolved as a dynamical systems theory whose emphasis is restricted to deterministic systems that exhibit sensitivity to initial conditions. The accepted terminology of nonlinear dynamics uses the word "chaotic" to mean "sensitive to initial conditions" [Strogatz 1994]. Thus the theory is often called "Chaos theory". However, I regard this choice of terminology to be unfortunate, in fact misleading - because of the strong association of *chance* with the word *chaos* in general usage of the term. One popular dictionary defines "chaos" as "where chance reigns supreme". However, *chance* is nowhere evident in, and is in fact banished from, nonlinear dynamics. Thus, *chance* most certainly does not "reign supreme" in nonlinear dynamics.

It turns out that *sensitivity to initial conditions* can sometimes lead to trivial results. Thus, some additional behavior is generally required by nonlinear dynamics to ensure that the systems it studies are interesting. So the theory is not quite as simple as "the study of dynamical systems in which a sensitivity to initial conditions is exhibited". Formally, a couple of other mathematical conditions are also imposed to ensure the richness of the class of systems studied by nonlinear dynamics. However, "chaos" ("sensitivity to initial conditions") is the central them, and generally characterizes the class of systems studied by the theory.

Another aspect of this theory is that dynamical systems that exhibit "chaos" must be "at least non-linear". Linear systems are never sufficiently complex so as to exhibit "chaos". Thus, the theory is well described as *nonlinear dynamics*.

Typically, the central interrelationship between and among the elements, or points, in nonlinear dynamics is proximity as measured by a metric on the space. These metrics are used to define rich and complex interrelationships and structure such as *fitness landscapes* and *basins of attraction*.

Also, typically, the "engines of motion", or dynamics, within these theories are differential equations. These may be either ordinary or partial, but in general their solutions are nonlinear and deterministic. This is true also of the equations of classical and Hamiltonian dynamics. However, the basic laws of Newtonian and Hamiltonian mechanics predetermine those equations. Nonlinear dynamics, on the other hand, is

less restrictive and explores more general, and more complex, possibilities for equations of motion, or time evolution.

At the risk of being overly simple, it is fair to describe nonlinear dynamics as a deterministic nonlinear time-invariant differential system that is sensitive to initial conditions.

How Organodynamics Portrays these Elements

We shall now introduce the basic elements of and approach to developing organodynamics – at least to the extent that it starts to become clear how organodynamics differs from the other dynamical systems theories already discussed, and to start to situate organodynamics within the pantheon of dynamical systems theories. The essential details of this theory, however, will be revealed throughout the remainder of this series of articles.

The purpose of organodynamics is to describe mathematically the changes that occur over time of how a complex system is *organized*. Traditional dynamical systems theories describe how individual particles change their location and velocity (or, optionally, momentum) over time. But, rather than focusing on the individual elements, from the beginning organodynamics emphasizes changes over time to a system-as-a-whole. (It can treat subsystems the same way.)

To do this, organodynamics creates a second-order entity called an *organization*. This entity represents how the elements of the system are *organized*, "arranged", or "configured" at some particular point in time. This *organization* is then traced over time to see how it changes. Each of these changes is treated as an instance of an *organization* entity. And the set of all of these changes in organization represents how the system itself *reorganizes* over time.

Thus, an *organization* in organodynamics and a six-dimensional vector in classical mechanics play the same role within their respective theories - the *representation of state*. This difference emphasizes the fact that whereas classical mechanics is primarily interested in the location and velocity of a particle, organodynamics is primarily interested in the *organization* of a system (or subsystem).

The notion of how a complex system changes its organization over time has significant meaning to a large class of complex systems - including a biological organism, the evolution of biological species, the molecular constitution of a chemical space in physical chemistry changes over time, human psychological development and learning, and many more exemplars.

In addition, the *changes in organization* that occur over time in many of these complex systems are *subject to chance variation*. That is, for any given time, there is more than one possible way that the system could potentially be organized at that time. And it is reasonable to say that each of those possible ways has a probability of manifesting at that time. In other words, the set of all possible ways that the system could be organized at a particular time is the basis for defining a probability space. We want to make sure that we include this chance aspect of *system organization* in the dynamical systems theory that we are constructing in these articles.

We can summarize what we have just said as follows: An *organodynamic system* is a mathematical abstraction that represents how a system of elements is organized and how that organization can change over time in a manner that is subject to chance.

In this section, as in the previous ones, we shall introduce how organodynamics treats the basic elements of a dynamical systems theory: elements, state, trajectory, state space, etc. However, the principle object of interest in organodynamic is the *organization* – which is a second-order object that is constructed from the elements of the underlying system. This fact creates additional complexity beyond that of traditional dynamical systems theories – a complexity that is intended to provide additional fidelity to models of the very complex systems that are targeted by this theory. Thus this theory needs elements that the others do not – which we introduce here.

To make sure that the reader does not get lost in these abstractions and lose a sense of relevance of the theory to practice, we shall use an example from physical chemistry that we shall refer to as "molecular compositional dynamics", MCD. By this we mean the typical situation encountered with a gas of mixed molecular type that is often seen in physical chemistry. More specifically, we mean a collection of atoms (and their ions and isotopes) of varying types within a closed region of space and with some fixed total energy.

So, in "MCD" we start with a set of atoms. But what we are really interested in is how the "chemistry of the situation" goes about organizing these atoms into some configuration of molecules; and then how that same chemistry (the dynamics) changes that molecular organization over time. For the purpose of this section, I shall treat this example from an intuitive perspective wherein atomic structures and related molecules compose and decompose over time to a chemical mileu in which the molecular structures change.

MCD then is an example that introduces the essential elements of organodynamics as a dynamical systems theory. We shall delve much deeper into these issues in Part II of this series, and use that analysis to ensure that our theory provides the required faithfulness to the kinds of complex systems that we want to model.

The System

In organodynamics, we begin with any finite (for now) set S of members, called "elements". These elements can be "arranged" or organized into a new structure or configuration that organodynamics calls an *organization*. Of course, there are many possible organization of an underlying set of elements. However, only one of them can be "realized" at any point in time.

However, it is these new constructs – these *organizations* – that organodynamics now switches its focus too. And it is the set of all possible organizations on this underlying set of elements that organodynamics declares to be the state space of its system-of-interest. Thus, each possible such organization of S is a potential state of this system of organizations. For this series of articles, a *system* is the state space of possible organizations of the underling set S.

For now, we shall also limit the cardinality of S to be finite, though typically incomprehensibly large. Moreover, the way that we are going to define how to determine the system – the state space – will confine it to also be finite – though incomprehensibly large. (Finiteness is a reasonable constraint here, because virtually all systems that we are interested in modeling with organodynamics have finite cardinality.)

In this *MCD* example, an element of S is any entity that is either an atom, an ion of an atom or an isotope of an atom. Collectively, we refer to these as *atomic structures*. The

set S, then, is the set of the atomic structures extant within the closed region being considered. Presently, organodynamics makes no distinction between the various types of atomic structures (as defined by the table of elements). This is a deficiency that needs further research.

An Organization of the System

The common idea of "organizing" a set of elements is to divide the set into a bunch of compartments; and then to allot each element to one of the compartments.

In our MCD example, we want to use the concept of "molecules" to do this. In MCD, the "elements" are atoms and their ions; while the "organizations" of S are molecules that contain atoms. In this view, the molecules "organize" the atoms into "compartments". And the particular way that all of the atoms are so organized by those molecules at a particular moment in time constitutes the "organization" of S by those molecules.

What we need to do now is to develop a mathematical construct that represents this kind of organization. The first step that we shall take to accomplish this is to define a bunch of subsets of S that reflect the way that the molecules in the MCD at one moment in time compartmentalize the atoms of S. Thus, for every molecule present in the space at time t, we define a subset whose elements are all of the atoms that participate in the molecule.

Now, some of the atoms of S will not reside in any molecule; and thus will not reside in any of these subsets. But, we want every atom in S to be covered by our subsetting scheme. So, lets create a new subset for each of those atoms – a singleton subset that contains only its atom.

At this point, we have a set of subsets of S that *covers* S. any *cover* of S gives rise to a topology on S – this topology consisting of all intersections and unions of the cover, plus the empty set. It is this topology that we define as an *organization of S*.

We are now in a position to define the *state space of S*. It shall be defined by taking all possible covers on S that represent chemically possible configurations on the given MCD, and then developing the topology that such a cover gives rise to. The set of all of these topologies on S is the state space of the particular MCD.

In a future article (Part IV), we shall elaborate this idea to an "extended topology". Until then, however, lets consider the *state* of an organodynamic system to be a topology.

State Space

The *state space* of S is the set of all permissible *organizations* of S.

In our MCD example, our state space is the set of all of the ways that one can form a collection of atomic structures into some molecules, but with possibly some of the atomic structures being "left out" – not involved in a molecule.

It is seen that a state space of S in organodynamics – being the set of all relations on S - is finite, but generally very large as compared with S. The state space is not ordered, and it does not (for now) have metric. But each of its elements (being a topology on S) can be very rich in its own right. These elements are NOT point-like "particles". However, this state space can be made to exhibit rich structure. For example, it is already a topology on S.

Time Domain

A *time domain* is a discrete or continuous set of real numbers that represent time points. In the present series of articles, we shall restrict our attention to discrete time domains. Discrete time points can either be fixed or stochastic. In order to simplify the exposition, we shall assume that they are fixed. However, it is straightforward to treat them as variable and stochastic, as is typical in the theory of stochastic processes. Even so, we shall leave the treatment of that case in organodynamics to later research.

Time Evolution as Trajectory

A trajectory in organodynamics tracks the *states* that an organodynamic system exhibits over a time period. This means that a *trajectory* of system S is a sequence of *organizations of S*. More formally, a trajectory is function whose domain is a time domain, and whose range is the *state space* of S.

A *trajectory*, then (for now), is a deterministic process – as in other DSTs. However, in other DSTs, a trajectory is a sequence of 6-dimensional vectors over time. But in organodynamics, it is a sequence of *organizations* over time. This determinism makes a trajectory useful in organodynamics for tracking the "path" of a system that exhibits deterministic behavior over time – or in tracking the path of a nondeterministic system after-the-fact – after realization. (Determinism is understood as the degenerate case of randomness, where entropy is zero.) Of course, it is not accurate to characterize trajectory in organodynamics as a "path", because it state space is not a metric space – and it most certainly is not a manifold as in classical dynamics.

Our MCD system provides a good example of what we have described so far. Our closed space of atoms and molecules has an initial configuration (*organization*) at the beginning of the time interval that we are observing. This constitutes its initial condition. Subsequently, the laws of physical chemistry determine the sequence of reconfigurations, or reorganizations, of this closed set of atoms and molecules over time.

Intimation of the Chance Nature of Very Complex Dynamical Systems

The history of science in the western world for the three hundred years has been overwhelmingly dominated by scientific determinism. Completely predictable precision has been the watchword. Little room has been made for any kind of fuzziness. A specified procedure given a specified input must always produce the same output. Chance variation found no home here.

But in the past 150 years, this rigid determinism has begun to buckle under the weight of the complexity of the systems that science has begun to study. First there was statistical mechanics, and then quantum mechanics. Fortunately, mathematics had begun to develop a rigorous treatment of probability, and these underpinnings came to the rescue of, while simultaneously being informed by, these bold, even outrageous, scientific disciplines of the early twentieth century.

Biological systems, by the last quarter of the twentieth century, are finally being widely recognized as the most complex dynamical systems in the universe. Even so, recognition of their chance nature was agonizingly slow to come. In the nineteen-fifties, the realization that chance operates at the foundations of biology was beginning to dawn on a few of the pioneering geneticists [Monod 1972]. Admittedly, those who had these insights at the time and voiced them encountered considerable intellectual

resistance. Fifty years later, however, probability and the theory of stochastic processes is well established in the burgeoning fields of computational and systems biology.

Of the many citations I could make from these early revelations, I will quote Jacques Monod, who was instrumental in the late nineteen-fifties in discovering the genetic apparatus that reads DNA and then synthesizes proteins by the operations of transcription factors, messenger RNA and Ribosomes. Monod recognized the prominent role of chance in the foundations of biology (esp. in genetics). And the examples that he cites reveal the range of the degree of chance variation that is expressed by various genetic mechanisms – all the way from maximal uncertainty to near certainty, or determinism.

Maximal uncertainty as expressed in the structure of DNA and reflected in the structure of the proteins that it specifies is evidenced in this quote from Monod [Monod 1972]:

> To say that in a polypeptide the amino acid sequence is "random" may sound like a roundabout admission of ignorance. Quite to the contrary, the statement expresses the nature of the facts.

On the other hand, the synthesis of proteins via the genetic mechanism ("gene expression") is near-deterministic:

> And so it must be acknowledged that the "random" sequence of each protein is in fact reproduced thousands and thousands of times over, in each organism, each cell, with each generation, by a highly accurate mechanism which guarantees the invariance of the structure.

It was perhaps unusual at the time for a scientist to attest to the continuum of degree of chance variation – preferring to understand "chance" as a binary condition, that things are either "random" or "deterministic", and never in-between. As we shall see, it was with the advent of information theory that a measure for the continuous degree of uncertainty of a probability space was formalized.

Accommodating the Chance Nature of Organizational Change

We have just described the general manner in which all of the elements that we are requiring of a dynamical systems theory are implemented in organodynamics. Unfortunately, a key aspect that we want organodynamics to exhibit is yet missing – *the chance nature of organizational changes over time*.

To remove any doubt as the centrality and fundamental nature of the role of *chance* in the dynamics of the most complex dynamical systems in the universe, lets begin this discussion with a citation from Jacques Monod – the Nobel laureate who was seminal in the discovery of how genetics operates in all living systems….

> We call these events ["noise" in DNA replication] accidental; we say that they are random occurrences. And since they constitute the *only* possible source of modifications in the generic text, itself the *sole* repository of the organism's hereditary structures, it necessarily follows that chance *alone* is at the source of every innovation, of all creation in the biosphere. Pure chance, absolutely free but blind, at the very root of the stupendous edifice of evolution: this central concept of modern biology is no longer one among other possible or even conceivable hypotheses. It is today the *sole* conceivable hypothesis, the only one that squares with observed and tested fact. And nothing warrants the supposition – or the hope – that on this score our position is likely ever to be revised. [Monod 1972, pp. 112-113]

> And so one may say that the same source of fortuitous perturbation, or "noise", which in a nonliving (i.e. nonreplicative) system would lead little by little to the disintegration of all structure, is the progenitor of evolution in the biosphere and accounts for its unrestricted liberty of creation, thanks to the replicative structure of DNA: that registry of chance, that tone-deaf conservatory where the noise is preserved along with the music. [Monod 1972, pp. 116-117]

This aspect is a significant differentiator for organodynamics. In the present section, we shall rectify this omission by adding four new elements – elements not needed in the traditional DSTs discussed above.

Lets first state that, with the addition of the new version of *trajectory* that we added in the previous section, we have managed to provide organodynamics with the equipment that it needs – up to this point – to be a deterministic dynamical systems theory. What we want to do in this section is to equip our *trajectory* construct with enough new probabilistic accouterment to convert organodynamics into a stochastic dynamical systems theory.

In order to do that, however, we must build up some mathematical machinery. We shall introduce these new constructs in an informal way at this time. Their more formal development must await subsequent articles in this series, especially Part V.

Strategy to Model Organic Systems as a Probabilistic Process

Recall that we have, in organodynamics, described a *trajectory* of underlying system S as a sequence of *organizations* of the elements of S, where the organization may change at any time. This is essentially a description of a deterministic system process whose states are *organizations*.

What we shall do in this section is to permit *chance variation*[3] to enter into this picture. Instead of being *certain* as to which *organization* of the elements of S will be manifest at any particular time point in the process, we shall allow any of a particular set of *possible organizations of S* to manifest at that point in time. Moreover, we shall assign a probability that it will manifest, or *be realized*, at this point in time to each of these possible organizations.

[3] I shall use the phrase *chance variation*, to describe the situation wherein any member of a sample space may be realized at the end of a statistical "trial". However, the phrase "stochastic fluctuations" is frequently used as an alternative. Another term often seen is "noise". I don't like the term "noise" for this usage because it implies that one of the sample points (the "signal") is "correct", while the others (the "noise") are erroneous. This interpretation is appropriate for some statistical applications (e.g. communications theory), but it is not in general the case. Rather, in general, all sample points in the sample space have an "equal right" to be realized in any trial, even while they may not have the same weight (probability). Statistical mechanics is a good example of this understanding. This use of "noise" for "chance variation" likely derives historically from the fact that Shannon introduced information theory in his article on communications theory. Besides, there is a propensity for many scientists to still believe that "the universe is deterministic", and that any use of probability merely reflects the observer's inability to discern "the truth".

In this way, we shall promote this idea of *trajectory* as a deterministic process into a *stochastic process, or probabilistic process*. It is this stochastic process of *system organizations* that will become our central construct in the modeling of organic systems. In the next few subsections, we shall give more detail as to our strategy for doing this.

System State as Uncertain

Lets begin with where we left off previously: the concept of trajectory in organodynamics. Recall that we developed the notion of *trajectory* to model *time evolution* within organodynamics. Our notion of trajectory represents a deterministic process expressed as a sequence of *system organization of S*, one for each time point in the process. In other words, each step in the process is a single system organization that describes how the system S is organized at that time point.

But, in organodynamics, time evolution is subject to chance. So, in order to introduce chance into this picture, we shall simply be less specific than this by saying that the *organization of S* that is actually realized for a given time step will be one of several possible such organizations. That is, we shall implement this lack-of-precision by saying that, for each time point, there is an entire set of organizations of S, of which exactly one will be selected. (For now, we shall say that the same set of organizations is available for choosing at every time point. However, later we shall permit the set of choices to change as time unfolds.)

At this point, then, we know – for any given time point in the process – that some organization of S will be selected from a specified set of organizations. But we don't know which one.

What we have described here is a *sample space*. That is, for each time point of the process, we have a sample space that consists of some organizations of S, exactly one of which will be realized at that time point. We shall call such a *sample space* an *organodynamics sample space*, or OSS. In these articles, for simplicity sake, we shall assume that all OSSs have a finite cardinality.

Each time point in the process can have its own OSS; or a single OSS can be used by all time point of the process; or anywhere in between these two extremes. In organodynamics, we shall often see the same OSS used for several contiguous time points, and then change. We call this a "piecewise homogeneous stochastic process."

The OSS is the first new construct that we need toward our task of defining a *stochastic process* that will represent chance-based time evolution in organodynamics.

Adding Probabilities

With the addition of organodynamic sample spaces (OSSs), we have added *chance variation* to our initially deterministic processes (which, before this, we called *trajectories*). However, In order to use probability theory, we need to assign probabilities to the sample points (which are *organizations*).

However, formally, probability theory requires a little extra work here. The formal construct in probability theory is called the *probability space*. A probability space consists of three parts:

1. The sample space

2. The probabilities
3. The sigma-algebra

The first and second of these are obvious. The *sample space* consists of the "things" that have probabilities. These "things" ("sample points") in the sample space are alternatives – exactly one of them can be *realized* at a time. And the probabilities are their probability assignments.

The "sigma-algebra" provides a mechanism for assigning probabilities to collective considerations – called "events". For example, suppose that we want to specify a probability value to the "event" that a game die landed with an even number of dots facing upward. Obviously, only one of a 2, a 4 or a 6 can land facing upward on any one toss of the die – because, being sample points – they are alternatives. However, we would like to be able to place a bet on the *event* that any of those will occur.

Therefore, we would like to be able to assign probabilities to events as well as to individual sample points. Such is the purpose of the sigma-algebra. It defines "events" as the logical combinations of the sample points.

For discrete probability spaces (with discrete sample spaces), the probabilities are assigned to the sample points, and the probabilities of the events are calculated therefrom. However for continuous probability spaces, difficulties are encountered when trying to assign probabilities directly to the sample points. For those, the probabilities are assigned to the events instead.

Since the probability spaces for organodynamics are always finite, then we shall assign probabilities to the sample points themselves (the *organizations*).

We have just added the next new construct that we need – a probability space whose sample points are *organizations of S*. We shall refer to such probability spaces as *organodynamic probability spaces*, or OPS.

Fortunately, in probability theory is it is customary to abbreviate a probability space with a construct called a *probability distribution*. In the case that the sample space is finite and all sample points have probability assignment, then is typical to use a kind of probability distribution that is called a *categorical distribution*, in which each sample point is associated to a probability. Also, most applications to which we apply this theory fit this case: they have finite, but very large, sample spaces. Thus, in these articles, when we use the phrase "probability distribution", we shall generally mean a *categorical distribution*.

So we have introduced one more new mathematical construct: the probability distribution, which acts as an abbreviated articulation of a probability space. If, as in our case, the sample space underlying that probability distribution is an *organodynamics sample space*, or OSS, then we refer to the probability distribution as an *organodynamics probability distribution* or OPD.

Time Evolution as a Stochastic Process

In the previous section, we developed the idea of an *organodynamics probability distribution* or OPD, to represent the chance nature of system organization at a single point in time.

What is left for us to accomplish is to represent the time evolution of an organodynamic system in a manner that embodies chance variation. We can now accomplish this

simply be defining an *organodynamics stochastic process*, or OSP, as a sequence of *organodynamics probability distributions*, or OPDs – one OPD per point in time, or "time step".

There are many ways that we can allot these OPDs to the various points in time across the time continuum. For example, we could use every point in the time continuum; we could use only a discrete set of randomly spaced time points; or, we could use only a discrete set of time points and require them to be equally spaced in time. All of these are viable choices – and should eventually be explored by this research.

However, these ideas are complex enough as they are. So, for the sake of keeping things as simple as we can for now, lets stipulate that, in the present series of articles, we shall take the later choice, and use only a discrete set of time points and require them to be equally spaced in time. We shall leave other possibilities for further research.

The OSP is the central mathematical construct in organodynamics. This structure has a great deal of flexibility. For example, we can "weave" multiples of these OPDs (by defining a set of n-ary operations on probability spaces) , each representing concurrent processes, into a web (an *organodynamic web*). The organodynamic web can then (theoretically) be used to model higher-level structures in complex dynamical systems - structures such as living cells or even whole organisms. This is accomplished by defining whole classes of "fork" and "join" operators, or transformations, on organodynamic probability spaces. So the possibilities within this kind of theory are elaborate.

A Stochastic Dynamical Systems Theory

We shall get into some of these developments in later articles of this series. But for now, lets summarize by saying that we can promote organodynamics to a stochastic dynamical systems theory by the addition of the constructs named *organodynamic state space* (OSS), *organodynamic probability space* (OPS), *organodynamic probability distribution* (OPD), and *organodynamic stochastic process* (OSP). At their foundations, each of these is established from a *state space* whose *states* are *organizations*, each of which are defined as an "extended" topology on an underlying set S of specified elements.

Time Evolution as a Dependent Stochastic Process

We shall make another refinement in order to accommodate realism. In "real life", it very often occurs that the "outcome" (which organization is actually realized for a time period) is affected – or "depends upon" – the actual outcomes of past time periods.

For example, if certain cards have already been played in a poker game, the likelihoods of which card will be actually dealt at the current time are affected. This is called *stochastic dependence*, or *statistical dependence*. *Stochastic dependence* is based upon *conditional probability* – as is *stochastic independence*.

For example, knowing the state (outcome) of today's weather my give you enough "extra information" so that your notion of the probability distribution that describes today's weather will be a different distribution depending upon the state of yesterday's weather. Such a situation is shown in the following matrix, called a *conditional probability matrix*:

Yesterday \ Today	Sunny	Cloudy	Rainy	
Sunny	0.0541	0.8108	0.1351	1
Cloudy	0.5814	0.1628	0.2558	1
Rainy	0.2500	0.5000	0.2500	1

Each row of this matrix represents a different probability distribution of today's weather, depending upon the actual outcome (realized weather) of yesterday. If yesterday was *sunny*, then the probability distribution for all three possibilities considered is in the first row. If yesterday were *cloudy*, then the second row would describe the distribution for tomorrow's weather, etc. (Notice that each row sums to 1.)

The fact that not all three rows are the same says that the two chance variables "yesterday's weather" and "today's weather" are stochastically dependent. That is, the appropriate probability distribution to use from the matrix "depends" the outcome of yesterday's weather.

On the other hand, if these two chance variables were stochastically independent, then all three rows would have been the same as each other, as shown below:

Yesterday \ Today->	Sunny	Cloudy	Rainy
Sunny	0.3200	0.4700	0.2100
Cloudy	0.3200	0.4700	0.2100
Rainy	0.3200	0.4700	0.2100

In this case, it doesn't matter which row one selects to determine the probability of today's weather, because today's weather is independent of yesterday's weather.

We can conclude from this that both stochastic dependence and independence are represented by the conditional probability distribution as articulated by the conditional probability matrix – with the special case of independence being represented whenever all rows are equal. Any other matrix – where not all rows are equal – are stochastically independent.

This conditional probability matrix can be derived from an associated joint probability distribution. To obtain the condition distribution from joint, simply divide each cell of the joint distribution by it row sum. It turns out that the resulting conditional distribution will be stochastically independent if and only if each cell of the joint distribution is the product of its columns sum and its row sum.

In other words, taking these dependency relationships across time periods into consideration actually affects the probabilities, and the OPDs. It affects them by making them more accurate. They are more accurate because extra information is taken into consideration in order to refine the results. In fact, this *extra information* takes some of the randomness out of the process.

Formally, the extra information *reduces the uncertainty* of the situation – of the stochastic process. This is evidenced by a well-known inequality from information theory: $H(X|Y) \leq H(X)$. In Part V, this inequality is explored deeper. In any event,

dependent stochastic processes generally reduce the amount of uncertainty of a stochastic process.

Many scientists are surprise at the assertion that chance can lead to stability. But many complex stochastic systems in nature exhibit this kind of stability [Austumian and Hanggi 2002]. A central theme of organodynamics is that there are conditions under which *chance phenomena* exhibit stable results. Discerning those conditions is a principle occupation of organodynamics. A principle preoccupation of information theory is to identify those conditions; and organodynamics shall leverage information theory quite heavily in this regard. In Part V, we shall see that these conditions have a lot to do with stochastic dependency.

Dependent stochastic processes are frequently encountered in applied mathematics, science and engineering. A special case of dependent stochastic processes is the Markov chain. For these reasons, organodynamics stochastic process (OSP) will most often take the form of dependent stochastic processes – a form that we shall call *organic dependent stochastic processes*, or *ODSP*.

Each conditional distribution can be mapped to a unique directed graph – or stochastic network. Such a graph has one node for each sample point. From each node, there is constructed an arrow to every node, including itself. Thus, the graph for the above conditional probability matrix has three nodes: one for each of sunny, cloudy and rainy. Also, each node has three edges (arrows): one drawn from itself to each node. This accounts for nine nodes. Of course, the conditional probability matrix has nine cells, each containing a probability. These nine probabilities are used to label each of the nine edges accordingly.

So, the conditional probability matrix has a network analog from. This means that any application that can be modeled using a stochastic network can also be modeled with a conditional probability. This fact avails the application of dependent stochastic processes to a large body of complex applications.

More Elaborate Changes

We have assumed above, for simplicity, that three particular aspects of an organodynamics stochastic process stay the same across all time points. These aspects are: 1) Change in the probabilities (OPD), 2) Change in the sample space of selectable organizations (OSS), and 3) Change in S. However, we know that all of these can change over time. Moreover, we need for the theory of organodynamics to accommodate all of these kinds of change.

In order to manage these possibilities within the theory, we shall treat the above kinds of change in an orderly manner. Specifically, we shall assume that: 1) A change in S generally induces change in the OSS and in the OPD. 2) A change in the OSS generally induces a change in the OPD. 3) A change in the OPD need not induce change in either the OSS or in S.

These consideration set up three levels of severity of change. In order of increasing severity, these are:

a. None of the above factors change over time. We have so far assumed this.

b. The OPD changes, but none of the other factors change.

c. The OSS changes. This induces a change in the OPD.

d. S changes. This induces a change in the OSS, which induces change in the OPD.

Strictly considered, any of the types of changes just mentioned would terminate an organic stochastic process (OSP) as we have initially defined it. But we want to allow these three kinds of changes to occur, yet still consider the resulting stochastic process to continue to "live" so as to be a model of more flexible and complex dynamical systems. Therefore, we shall allow such changes generalize our definition of OSP to include these kinds of changes. Essentially, these generalizations will formally promote our OSP into the *piecewise homogeneous stochastic process* that we introduced earlier.

Information Theory – the Mathematical Foundation of Organodynamics

The previous section has introduced a number of levels of flexibility – and uncertainty - into our model. The discipline that studies uncertainty in probability spaces is *information theory*. Its basic unit of measurement – or "statistic" – is that of *statistical entropy*, which we shall also call *stochastic entropy*, or just *entropy*.

Information theory defines an entire arsenal of measures, all of which in some way describe the amount of uncertainty inherent in various renditions of probability distributions. Collectively, these measures are called *entropic functionals*, and include entropy, joint entropy, *conditional* entropy, relative *entropy*, *mutual information*, and *entropy rate*. These functionals are very useful in imposing structure on probability spaces and stochastic processes.

Technically, *information theory* is *the study of entropic functionals*. Informally, information theory is the study of *uncertainty* as measured by entropic functionals. Says mathematician Richard Kleeman [Kleeman 2012], "The central idea of information theory is to measure the uncertainty associated with random variables."

Claude Shannon [Shannon 1948] characterized *entropy* as a measure of *choice*, *uncertainty* and *information*. An investigation into how entropy can measure all of these (apparently) disparate concepts at once leads one to an appreciation of just how flexible and powerful the tools of information theory can be in modeling stochastic dynamical systems.

The previous section mentioned piece-wise homogeneous stochastic processes. Such structures reference multiple probability spaces – or a space of probability spaces. Such a space can be structured by the imposition of some of these entropic functionals. For example, *relative entropy* is an entropic functional that supplies a kind of quasi-metric on a space of probability spaces. While *mutual information* measures the degree of stochastic dependence between two variables – another quasi-metric.

Unfortunately, the phrase *information theory* has undergone semantic inflation over the years (since 1948 when Shannon invented it), and has come to include *communications theory* – which is usually understood as a branch of electrical engineering and computer science. But, I argue that *communications theory* is the study of *message systems,* whose elements are senders, receivers, messages, etc. But this makes communications theory an *application* of information theory, rather than a part if it.

I see information theory as "the study of the uncertainty of probability spaces as measured by entropic functionals" – and thus a branch of probability theory. Thus I see information theory as a branch of mathematics, not computer science, electrical engineering or "information technology".

The “mistake” that has been made (in my opinion) in conflating information theory with any of these other disciplines is like understanding calculus as a branch of physics. Just because calculus was initially invented to solve physics problems doesn’t restrict it to being a branch of physics. Fortunately, Newton realized that. Just so, information theory – the study of uncertainty [Kleeman 2012] – is far more applicable than merely to “the mathematical theory of communications” (the title of Shannon’s paper). In Part V we shall use information theory (entropic functionals) as the mathematical foundation of organodynamics.

Organodynamic Systems

In this article, we have identified a new class of dynamical systems that we intend to study. Our motivation for singling out this class is to be able to emphasize a different idea of dynamical *system state* from the one that is emphasized by traditional dynamical systems theory.

Traditional theories are primarily interested in the dynamical state (trajectory) of individual system components (i.e. particles). And their notion of state is strictly physical. Specifically, their notion of state is defined by the position and velocity of those particles. Some versions (Hamiltonian) substitute momentum for velocity to great advantage. Nevertheless, their concerns for the notions of *state* and *trajectory* are strictly physical.

Of course, that is appropriate for physical systems. But we understand the notion of dynamical systems in a broader sense – a sense in which a dynamical system is viewed from a perspective that is not necessarily physical in nature. In fact, the system need not be physical – as long as it has a notion of *state* that changes and some notion of dynamics – understood as interrelationships between past and future state.

In particular, in organodynamics our interest in system state is in *the way that the system is organized*, and in *how that organization changes over time*. In some very complex systems, for example, our interests extend beyond the physical to the organizational. A conspicuous example of this is biological systems, living systems.

While ideas of position and velocity (or momentum) clearly apply to biological systems, our interests in those systems generally move beyond those physical issues. Instead our interest in biological systems immediately moves to issues such as metabolism and reproduction in individual organisms and to evolution and ecology in whole species. But metabolism, reproduction, evolution and ecology all pertain to the idea of *system organization*. In fact, they each can be understood as studies in system organization.

But our interests in system organization are not confined to static relationships. Rather, we are immediately interested in change of system organization, and what makes that change “tick”. This is dynamics – organizational dynamics.

And there is one aspect of the organizational dynamics of our target class of complex systems that organodynamics emphasizes: that, due to their complexity, these systems are generally *subject to chance variation*. Chance variation is evidenced whenever the same process (procedure) operating with the same initial conditions (inputs) does not consistently produce the same outcomes (outputs).

Chance variation in complex dynamical systems results in those systems being generally unpredictable. However, this chance variation need not be totally random, and also it need not rule out instances of complete determinism. Thus, the nature of

the observed chance variation in these kinds of complex systems is that it occurs in various degrees – degrees of uncertainty.

In other words, the dynamics of these kinds of systems is interplay between the past and future interrelationships of a systems state with some degree (large, small or intermediate) of chance variation, uncertainty, randomness or indeterminism.

From this discussion, we can see three principles of organization at work. We shall now list these. Further, we shall identify all systems that exhibit these three properties as a class – a class of complex dynamical systems that we shall name *organodynamic systems*.

An organodynamic system is one that exhibits the following three *systemic properties*, which serve as *organizing principles*:

Organization. The system is defined as being constituted by a specific set of elements. Moreover these elements exhibit certain interrelationships, which are also defined. In addition, the elements may be apportioned across a set of compartments. These compartments and their apportionment are also defined.

Reorganization. Over time, the organization of the system is subject to change. There are past-future interrelationships among the system states that influence which organization states are realized at each time step.

Uncertainty. These past-future interrelationships among the changing system states over time are subject to chance variation. The nature and degree of this chance variation can characterize and qualify which system states are realized at each time step.

Organic Complex Systems

While *organization*, *reorganization* and *uncertainty* are the central themes of organodynamics, there is a lot more to the class of highly complex systems – such as biological systems – than is captured by these three systemic properties alone.

It is understandable that life scientists understand biological systems in terms of "What are they made of?", or their constitution. Biologists are first and foremost empirical scientists. When they observe biological systems, the first thing they see is a conspicuous participation of CHONPS atoms[4]. So it is reasonable that biologists define life in terms of its constitution - essentially, as "carbon chemistry".

Systems theorists have a different perspective - they look at any system with the eye to "What are its systemic properties?" What the system is made of is secondary to a systems theorist. In fact, it is desirable if the elements of a system are interchangeable – as long as they are capable of preserving the systemic properties.

For example, in object-oriented systems design, the "constitution" of a system is encapsulated behind an "interface" which exposes the behavior (dynamical systemic properties). This behavior is exposed to the external environment outside of the system, while the encapsulated constitution remains hidden and is considered an "implementation detail". Engineers are free to exchange one "implementation" for another – as long as the systemic properties that are exposed to the outside world are preserved. When engineers "gather requirements" for a system that they are about to build, what they are doing is assembling a list of systemic properties that must be met

[4] Carbon, Hydrogen, Oxygen, Nitrogen, Phosphorus and Sulfur.

and exposed to the world external to the system. While designing the system, they decide on the best constituents to use in order to achieve those properties. But the constituents are “merely an implementation detail”. The engineers are typically free to make those choices in any manner they choose, as long as they optimize the required systemic properties.

So, as a systems theory, organodynamics’ understanding of how a class of systems is defined is by providing a list of systemic properties. Any system that exhibits those properties qualifies as a member of that class of systems.

The three organizing principles that we have already put forward are part of this list. But this short list is not enough to capture the salient nature of living, or even life-like, systems and requires some embellishment. Of course, it is not our intention here to define “life”, but rather to describe a rather large class of very complex systems that – if the theory is worth its salt – must by necessity account for life and even the life-like as well, owing to their complex dynamical natures.

Thus, at this time, we shall identify four other salient features of the class of complex systems that we are interested in. In fact, we shall define this class of interesting systems is by whether they exhibit all seven of these systemic properties.

To guide us in our selection of these other properties, we have used biological systems as an inspiration. Certainly any class of very complex systems must include the living. So we looked at biological systems and asked what other systemic properties they exhibit that makes them a very interesting class. We then added these properties to the list. Essentially, we asked “What are the essential systemic properties that a dynamical system would exhibit that would argue for characterizing it as “lifelike” – regardless of its constitution.

Of course, the answer to that question is governed by one’s sense of taste. So, admittedly, I have selected these additional traits based on a specific set of interests.

Nevertheless, these seven systemic properties *do* define a specific class that includes any systems that exhibit all seven. Whether these are found to be interesting is for the reader to decide. I do not intend for these seven properties to *define* life, but rather to be inclusive of biological systems.

We shall call this new class of systems - that exhibits the seven properties that I am about to list – *organic complex systems*.

Obviously, the class *organic complex systems* form a subset of the class *organodynamic systems*. We have troubled ourselves to define both classes because we believe that the applications call for it. In fact, there are some not-so-complex systems that are *organodynamic*. (This fact means that there are some relatively simple examples of organodynamic models.) However, as we shall shortly see, it is difficult to imagine an *organic complex system* that would not be universally understood as complex.

Without further discussion, let me list the seven systemic properties that characterize the class of systems that we shall call *organic complex systems* (OCS).

An organic complex system is one that exhibits the following seven *systemic properties*, which serve as *organizing principles* (including the tree that we have been dealing with since the beginning of this article):

Organization. The system is defined as being constituted by a specific set of elements. Moreover these elements exhibit certain interrelationships, which are also

defined. In addition, the elements may be apportioned across a set of compartments. These compartments and their apportionment are also defined.

Emergence. All organic systems exhibit at least one systemic property that none of its components exhibits. This is called *systemic emergence*.

Compositeness. All organic systems are "nested". That is, they have at least one component that also has its own components and their interrelationships. This nesting may occur to any finite number of levels of organizational depth.

Reorganization. Over time, the organization of the system is subject to change. There are past-future interrelationships among the system states that influence which organization states are realized at each time step.

Uncertainty. These past-future interrelationships among the changing system states over time are subject to chance variation. The nature and degree of this chance variation can characterize and qualify which system states are realized at each time step.

Autocoorganization. How do organic complex systems accomplish the reorganization of their components in such a way that all seven of these systemic properties are preserved? The answer is that each component participates in reorganizing the other components - resulting in a reorganization of the system. Of course, it is the specific interrelationships among those components (their *organization*) that enable them to change their organization in such a way that the seven systemic properties of OCS have a sustaining probability of being preserved in the resulting reorganization. Thus, it is reasonable to say that the system has a propensity to continuously reorganize itself over time through its components and their relationships in a manner that preserves all seven systemic properties of OCS.

Persistence. Persistence is the continued exhibition of all seven of these organizing principles by an organic complex system. The nature of the autocoorganization and uncertainty of an organic system is that it engenders the limited ongoing existence of these systems in a manner that preserves all seven principles of organization. Taken together, all of these static and dynamic organizing principles can result in, at least, a limited persistence of organic complex systems. Once any one of these principles fails to be preserved, then the organic system ceases to qualify as an organic complex system.

The reader will notice that above list of systemic properties that defines *organic complex systems* is organized so that the *static properties* (first three) and *dynamic properties* (last four) are placed together. (Most systems theorists define *emergence* as a dynamic property. We admit to that meaning and label it *process emergence*. Our usage however is static. We label it *systemic emergence*.)

Ultimately, a chief responsibility of organodynamics as a systems theory is to provide a mathematical foundation and articulation for the seven concepts put forth in the present section. It is the intention that the remainder of this series of five articles will accomplish that.

Objectives

Now that we have introduced what we mean by organodynamics and organic complex systems, it is time to state what we want to accomplish in this series of five articles. Our objective is to provide enough acquaintance with organodynamics that the reader

will be encouraged to participate and perhaps collaborate with the further development of this theory. We shall proceed to do this by sketching how organodynamics was developed. The last article will more formally describe the mathematics of organodynamics.

We have taken an empirical approach to developing organodynamics. We initially encountered a complex system that we desired to model. Specifically, it was the development, maintenance and operation of a suite of interoperating distributed enterprise-computing applications, each of which was distributed across multiple data centers that spanned thousands of computers. Later, we sought out complex systems in nature that shared similar properties.

An analysis of these systems revealed extraordinary dynamical structure and systemic properties. I searched for a systems theory that exhibited the same properties to use as a foundation for constructing a model of these systems, but found none that were satisfactory. Always there were significant systemic properties that were not supported by these theories. Subsequently, I surveyed a number of natural systems and found similar properties, but even more complexity. In this way, I encountered the limitations of prevailing dynamical systems theories to support an entire class of complex systems. From these observations, I began to construct a new dynamical systems theory – organodynamics.

In the next three articles, I want to sketch our analysis of these complex systems and briefly reconstruct the systemic properties that I found consistently exhibited across this class. This investigation will reveal what is needed in order to develop a general systems theory like organodynamics.

Preview of Future Articles In this Series

The present article in this series on organodynamics has described the objectives of the theory as an attempt to provide a new dynamical systems theory that focuses on modeling certain types of very complex systems. This article has tried to situate organodynamics against a background of existing dynamical systems theories; and has attempted to differentiate it from them and to identify its intended domain of application.

In the second article of this series, *Part II: Analysis*, I shall present a relatively simple exemplar application in some detail, and show how the various constructs and mechanisms that were introduced in Part I play out when applying organodynamic theory to a physical application. In this instance, the application is taken from physical chemistry.

In the third article, *Part III: Further analysis*, I continue our examination of exemplar complex systems to see what else we need to add to the theory, and to see how the construct of the theory work with a more complex – and more biological, exemplar system. This time, our exemplar is human cognition. This will allow us to stretch capabilities of the theory beyond the simpler physical chemistry example presented in Part II.

In *Part IV: Prediction Dynamics*, I confront a subject that I have been alluding to since Part I, but have not yet addressed formally. This is the issue of “Can a theory based on chance and probability theory be expected to do anything other than degenerate into unpredictable chaos?” And, “Are there are any mathematical conditions under which such a foundation can be expected to become well behaved – at some point in time?”

If so, then the future behavior of these systems should at least be bounded or constrained. There should be "at least something" that can be said about the time evolution of these systems. And we should be able to make some kind of predictions as to how they might be expected to play out. In other words, organodynamics needs to present a stronger kind of *dynamics* than merely that "the time evolution of these systems is subject to chance". Part II addresses questions and sets a direction for their answers.

Part V takes us beyond predictability and into the realm of complex systems that cannot afford to exhibit consistent predictability if they want to persist (survive). Biological life exemplifies this trait through adaptation via diversification. *Part V: Autocoorganization* addresses the issue of defining a stochastic dynamics that can result in adaptive behavior, yet still be defined strictly in terms of probability, information theory and stochastic processes.

I invite you to read these articles and to engage in the creative discovery of mathematical and systems-theoretical mechanisms that can bring this theory to life.

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