# Organodynamics: A General Theory of Dynamical Systems based on Chance Organization

## Part II of V: Analysis

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## Introduction

This is the second article in a series of five that introduces a new dynamical systems theory named *organodynamics*. The first article, Part I, describes a class of complex dynamical systems for which this theory seeks to provide a modeling platform.

Conspicuous and salient features of the members of this class are 1) the ways that complex dynamical systems organize their members, 2) the ways in which they change their organization over time, and 2) the chance nature of organizational change within those systems. We named these three salient features "organization", "reorganization" and "uncertainty". Part I also explains that the need for a new systems theory derives from the fact that no existing dynamical systems theories are known that emphasize these three salient features, and that use them to define the concepts of *state*, *trajectory* and *dynamics*.

Part I also outlines a mathematical strategy for developing organodynamics into a dynamical systems theory that provides formal constructs for modeling these three central ideas, and thus offers a modeling platform for this class of dynamical systems.

However, in order to test whether such a mathematical strategy can adequately provide such a modeling platform, it is necessary to scrutinize more closely a principle exemplar system of the target class. Engaging in this scrutiny is the purpose of the present article, Part II.

We shall revisit an example cited in Part I – a dynamical system from physical chemistry that we dubbed "molecular compositional dynamics". We shall inspect this system asking 1) "Under closer inspection, does the mathematical strategy we laid out in Part I appear to hold", and 2) "Has this scrutiny of this exemplar unearthed any other salient features that must be accounted for by organodynamics, in order to properly support *organization*, reorganization and *uncertainty*?"

## Analyzing a Primary Exemplar System

One of the complex dynamical systems that have inspired the quest for a new modeling platform is one that we are calling *the molecular compositional dynamics environment*, or *MCD* for short. We alluded to this system in Part I, and used it there as a preliminary example.

However, in this article, we want to examine it more closely to see if the mathematical strategy that we outlined in Part I for developing organodynamic holds upon closer

examination. We also want to start to build the mathematical constructs that constitute the theory of *organodynamics*, many of which we alluded to in Part I. We shall initiate that development in this article.

## *Molecular Compositional Dynamics (MCD)*

By "MCD environment" we mean an environment of atoms and related ions of various isotopes of the atoms that occur in a closed molecular environment. Our focus in MCD is, given a closed system of atomic structures, what are the various ways that those atoms can be organized in to molecular configurations; and how can those atoms consecutively reorganize into many of these possible molecular organizations over time. In other words, we are primarily focused on molecular organization and change in molecular organization. We shall not represent molecules as point "particles", but rather as *organizations* of simpler elements – namely atoms and other smaller molecules.

We begin by viewing a closed chemical system as a set of atomic structures (atoms and their ions, etc.) that, over time, chemically combine and recombine to form chemical various chemical compounds (simple and complex molecules). What we are focused on is the composition of all of the molecules in the closed space at any point in time, and how those compositions change over some time period.

The dynamics of such an environment is characterized by the various chemical reactions that can occur. Such an environment is reminiscent of the "ideal gases" of statistical mechanics. However, those environments were intentionally chemically simple, generally consisting of a single atomic type, and with no consideration of the formation of molecules through chemical reactions. In organodynamics, our notion of "MCD" generally sports multiple atom types as well as a rich molecular mileu that results from chemical reactions against the initial space of atoms.

One often sees definitions of *molecular dynamics* that focus on the *rates of chemical reactions*. However, these *rates* are merely one narrow focus on the very complex and dynamical environment that underlies MCD as I am defining it. The normal definition of *molecular dynamics* focuses narrowly on rates of change, and misses the actual changes of molecular organization and reorganization – the sequences of actual molecules formed over time.

Of course, a "rate" is a number, whereas a change in molecular organization is a rich and complex configuration or arrangement of multiple atoms – and is therefore best represented as a pattern of organization of atoms. Representing change as a simple number is comparatively easy; and a *pattern of organization* is more difficult. The latter is what we seek in organodynamics. As we shall see, I will use the mathematical construct of a *topology* to represent such a pattern of organization – whereas traditional *molecular dynamics* is confined to using dimensionless real numbers to measure reaction rates. Obviously, a topology is capable of capturing considerably more complex relationships than a single number.

This environment – if not reduced to a one-dimensional focus on "rates" – is an immensely rich and dynamic system whose organization is of considerable complexity, and whose dynamics is driven by varying degrees of chance variation in the form of chance encounters of chemical entities. In other words, the MCD environment is the "poster child" for Organodynamics – owing to its complexity in both areas of *organization* and *chance variation*.

The complexity of the notion of *organization* goes in two directions within the environment of MCD. In the first place, this environment can be viewed as organized units (molecules) that are embedded within each other to many levels of organization. And secondly, these organized units, or *organizations*, are constantly changing, or reorganizing. This is true on the level of individual units as well as on the level of entire embedded structures.

In addition, the notion of *chance variation* within this environment is also exceedingly rich. Of course, one must first identify exactly what entities are being singled out as exhibiting chance variation. Intuition would suggest that it is the individual chemical units (atoms, molecules, macromolecules, etc.) whose variation would be of interest. However, Organodynamics happens to be more interested in *how these individual chemical units are organized*. And that it is those organization units (molecules) that are subject to chance variation – as the chemical mileu organizes and reorganizes over time through chemical reactions and interactions.

In other words, Organodynamics is interested in "all of the possible collections of molecules that can be constructed from a specified set of atoms" more so than simply what or where the atoms are. So, from a chance variation perspective, Organodynamics is centrally interested in the probability distribution of the various ways that chemical entities can combine into bigger entities – entities of a higher level of organization than their elements.

MCD is complex, so we shall develop a model of it incrementally as a series of "approximations". Our first approximation will develop a model for the elements of MCD. Our second model will embellish our element model. Subsequent approximation of our MCD model will address various aspects of dynamics, including state change, time evolution and dynamical constraints.

## *First Approximation of MCD: Atoms and Molecules*

The central purpose of this Part II is to subject the MCD exemplar system against the definition of organodynamics that I put forth in Part I to see how well that definition is a able to model MCD. So we need to specify the elements of MCD that correspond to the following concepts for this model: elements, underlying set (of elements), organizations, state, state space, trajectory, etc. In this section, we begin our specification of what we are going to choose within the MCD environment as the *underlying set of elements* S, and how we are going to create *organizations* of S for MCD.

### MCD Elements and Organizations

Lets begin by saying that, in MCD, we are starting with an *underlying set* of *atoms*, and we want to organize them into a set of *molecules* – including small, medium and very large molecules of complex organization inner structure. It is fitting, then to decide that the *underlying* set construct in organodynamics will map to the concept of the set of atoms in MCD. Moreover, the concept of organization will map to the concept of *molecule* in MCD. Since both molecules in MCD and *organizations* in organodynamics can be arbitrarily complex, then this modeling decision is reasonable.

These are the basics – an underlying set of elements and a structure that represents ways that those elements can be organized - that we need to define model MCD using the mechanisms of organodynamics. To complete this model of MCD, then, we shall

now discuss how we shall define the other mechanisms of organodynamics within the realm of MCD: state, trajectory and dynamics.

## State

We have said that organodynamics is different from traditional dynamical system theories largely because of how it sees *state change*. Whereas, traditional theories are focused on the *location* and *momentum* (or velocity) of a *particle*, organodynamics is focused in the *organization* or *structure* of *system*.

This change of focus and interest has its downside, because *system organization* can *extraordinarily complex* to describe as compared to *location* and *momentum*. But this fact is one of the things that make complex systems complex. We don't want to have to refuse to model what we are interested in just because our modeling technology isn't complex enough to do the job.

Therefore, our mechanism for representing *system state* in organodynamics will have to be *complexified*. In this section, we are going to present the constructs that we intend to use in organodynamics to represent *system organization* as *system state*.

There are two aspects to system organization that we want the structures that we are going to present here to capture regarding system state. These are:

1. How the system's organization allocates its elements into "compartments".
2. How the elements of the system are interrelated.

These two concepts embody two different dimensions of "organizing" the elements of a whole: 1) how the elements relate to the system as a whole, and 2) how the elements relate to each other. Both are reasonable notions of "organizing" a set of components; and I want the mathematical construct that is used to represent an *organization* to be able to capture both of these ideas simultaneously.

In addition, we want to be able to represent two levels of complexity: *simplex state* and *composite state*. *Composite state* will provide for "system nesting", where in smaller systems can be understood as – in some sense – members of larger systems. Simplex state represents two levels of organization within the underlying system being categorized. In our MCD example, simplex sate can model a space of atoms whose molecules consist of just atoms. But complex state can model a space of atoms those molecules might consist of other molecules or of atoms, or both.

Of course, *simplex state* is the special case of complex state where these multiple levels of organization are not occurring. That said, if I only define composite state, we get simplex state as a special case. So all I have to do is to define organization in a sufficiently general way that it includes both simplex and complex state, and I can then do away with the distinction.

The structure that we are going to define in this section that implement these ideas and relationships should enable the simplex case to be a special case of the composite case. Admittedly, the resulting structures will be somewhat complicated to describe. But extensible and elegant machinery is needed here. Very complex systems require a complex theory to model them. On the other hand, the simplex cases should end up being a simpler articulation. The goal in developing organodynamics is to provide enough complexity that enables the modeling of very complex systems – and

represents all of the aspects of interest; but at the same time provide enough elegance so that the simple case are modeled relatively simply.

As well, organodynamics needs to provide modeling guidelines, in the form of a *modeling methodology* that assists modelers to start simple and take an incremental approach – thus building gradually to more complex models. (The exposition of such a methodology is beyond the scope of this series of articles.)

In the next couple of subsections, we shall use our MCD example to develop the necessary mathematical constructs to be able to model *system organization* as *system state*, and to support both a simplex and complex (composite) version of that state.

Nevertheless, we shall develop simplex and complex states separately, so that we can introduce them incrementally – and thus gradually. In the end, though, a single definition will cover them both.

## Implementing Simplex State

To represent simplex state, we need to define a mathematical construct that captures both of these ideas:

1. How the system's organization allocates its elements into "compartments".
2. How the elements of the system are interrelated.

We shall do this by 1) using the construct of a topology to accomplish compartmentalization, and 2 using the construct of a set-theoretic *relation* to accomplish element interrelationships. I shall discuss each of these in turn in the next two subsections.

## Use Topologies to Represent Compartmentalization

For MCD, define S to be the set of all atomic structures in the MCD environment. Of course, the current molecular configuration of S has the effect of organizing these atoms into various compartments. We shall show now how a particular topology on S can capture this compartmentalization. And we shall use such a topology to define an *organization* of our MCD system.

We first note that the current molecular organization of the atoms of S from a *cover* of *some* of the atoms of S. The atoms that are *covered* are the ones that currently participate by being involved in some molecule of the current molecular organization of S. Of course, some of those atoms (ions) are shared covalently between two or more such molecules. Nevertheless, the collection of all of those molecules provides a cover of all of the atoms of S except any "free" atoms that not currently participating in some molecule.

But we want for all molecules to be covered. The reason is that: 1) I am trying to generate a topology on S that I can use it to define an organization of S, and 2) I can easily generate such a topology from a cover of S.

I can create such a cover by starting with the one created by all of the current molecules of S and making sure that all of the "free" atoms are also covered. This is easy to do by creating a singleton set for each "free atom" and then adding it to the above cover. Now, this set of subsets of S does cover S.

I will then generate a topology, $T_S$, on S by creating all of the (finite) unions and intersection of the subsets in this cover. If I add the empty set (if necessary) to the resulting set of subsets of S, then I will have a topology $T_S$ on S that represents the current molecular configuration of S.

This topology, $T_S$, captures the way that the current molecular organization of S "compartmentalizes" all of the atoms of S. In that sense, this topology represents the organization of the elements of S by the compartmentalization of the current molecular organization of S. Each open set of this topology represents either 1) a molecule of S, 2) the shared atoms of several molecules of S, or 3) all of the molecules that lie within some subset of molecules of S. Each of these is worthy of being considered as some "compartment" within S that is worthy of distinction. Thus, as far as I'm concerned, this topology is a useful way to compartmentalize S.

Thus, we have developed a mathematical construct, the topology $T_S$ on S, which captures that kind of organization of S.

## Use Set-theoretic Relations to Represent Interrelationships between Atoms

Interrelationships among entities form directed network relationships. We can represent this mathematically in a number of ways. One obvious way is the use of a directed graph from graph theory. This is a good candidate.

Another way is to use the set-theoretic concept of a *relation* on S – in the sense of a set of ordered pairs of elements of S. This choice is easy to articulate. So, for now, we shall leverage it to represent our second idea of "organization". In other words, we shall use a set of ordered pairs of S (a *relation*) to represent the directed relationships among the elements of S.

This approach has its limitations. Specifically, multiple relationship types are not distinguished by this approach. However, the search for alternatives will be left to further research.

The question that next arises is "How can I combine these *relations* with the topology concept developed above. This will be answered in the next subsection.

## Integrating the Topology and the Relation into a Single Organizational Construct

I shall unify these two concepts of system organization – the compartmentalization and the interrelational – by starting with the compartmentalization (the topology) and adding the interrelational (the relation) to it in a certain way.

The members of the topology, of course, are all subsets of S that have some atoms as their elements. These members are called "open sets" in topological jargon. What we are going to do is to associate each of these open sets with a *relation* on the atoms that open set. Recall that this relation describes a certain interrelationships that the atoms of this open set have among themselves, if any.

So, at this point, we have all of the open sets in the topology $T_S$ associated, or paired with, a relation (set of ordered pairs) that describes how the atoms in that open set are interrelated. These associations form pairs whose first element is an open set of the topology $T_S$, and whose second element is a relation on the atoms within that open set.

What we are going to do next is to form a set of all of those pairs. We shall call this set an *extended topology* on S, symbolized by $T_{RS}$. In other words, an *extended topology*

$T_{RS}$ on S is a set of ordered pairs whose first entry is an open set of topology $T_S$ on S, and whose second entry is a relation on that open set:

$$T_{RS} = \{ (O, R ) \mid O \text{ is an open set in } T_S, \text{ and } R \text{ is a relation on that open set.} \}$$

Thus, an instance of $T_{RS}$ represents the *organizational state* of an organodynamic system.

Of course, there are several possible such states. In fact, any particular molecular organization that is possible for S is represented by one of these $T_{RS}$ organizational constructs.

### The Space of All Possible Organizations of S

Next, I want to consider the space of all possible ways that S can be organized – that is, the *state space* of S. We shall do this in the present section.

And we shall be eventually interested in how "S changes its organization over time". For such a consideration, there must be the idea of the "set of all possible ways that S can be organized". This implies that there exists a "space of all possible organizations of S". Such a space is, in fact, the set of all possible $O_{TRS}$, as defined by the various ways that S can be organized into different molecules – and this organization, of course, depends upon the chemical bonds among the atoms at some point in time.

That is, we shall define the "space of all organizations of S" as

$$\mathbf{O_S} = (O_{TRS} \mid O_{TRS} \text{ is an organization of S}\}$$

A complete description of an organodynamic system should associate, or pair, and underlying set S with $\mathbf{O_S}$, its "space of all possible organizations of S". This we do with the pair (S, $\mathbf{O_S}$).

This completes our description of the mathematical equipment that we are providing to represent *simplex system state* in organodynamics; where, by "simplex" here we mean "not nested or composite".

In the next approximation of the theory of organodynamics, we shall create the additional machinery to represent *composite systems*.

## Second Approximation: Compound Molecules

So far, we have viewed a molecule as being constituted by elements that are *atoms*. However, some molecules are very complex, and are usually by chemists as consisting of other, smaller, molecules. For example, *proteins* are complex molecules that are usually understood as a chain of smaller molecules called *amino acids*. And *DNA* is a complex molecule that is usually understood as a chain of *nucleotides*.

To accommodate this kind of structure, this section introduces the idea of molecules whose elements are either other molecules. The situation is sometimes even more complex than this, because some complex molecules, in addition to being made up of smaller molecules, sometimes have some "extra atoms" involved as well. Thus, we want to include in our model a structure that is capable of representing complex molecules whose elements are either other molecules, or atoms or both.

We shall refer to such molecules as *composite* molecules. In this section, we shall develop the necessary mathematics to represent composite molecules in organodynamics.

### *Molecules of Molecules*

Lets begin this consideration with a complex molecule that is quite a bit less complex than a protein or a DNA molecule – but that still has elements that are other molecules nevertheless.

Consider for example a molecule named *ceric sulfate*, whose chemical formula is $Ce(SO_4)_2$. It is reasonable to say that ceric sulfate is a molecule that has three elements. They are one cerium atom (Ce) and two sulfate molecules ($SO_4$). In turn, each of its smaller molecular elements has *atoms* as its elements.

So, ceric sulfate is a composite molecule that has three molecular elements, two of which are small molecules, each having its own atomic elements. In addition, ceric sulfate has another clement (Ce) that is already an atomic structure. Thus, the elements of ceric sulfate is mixed: two smaller molecules and one atom.

This notion of *composite molecules* allows us to represent very complex *organization* entities within organodynamics. And, since it is organization entities that represent *state* in organodynamics, then we can have very rich state. This implies that the notion of *state space* in organodynamics – being the set of all possible *organizations* of the elements of an underlying set S. Clearly, *composite molecules* represent “multiple levels of organization”.

The reader is reminded that (for now) we are limiting the cardinality of our underlying sets to be finite – though usually large. If one imagines the set of all possible organizations on S, it can be seen that such a set can be very large. For MCD, this *state space* represents the set of all possible molecules – some simple, some composite – that can logically be constructed from an initial underlying set of atoms. Now that we have added the possibility that this space can also contain *composite organizations*, such as the compound molecules we have just introduced, then it can be seen that such a space can be exceedingly larger – though still finite. SO, even though our state space is finite, its members are individually rich and complex.

The compound molecule *ceric sulfate* is somewhat interesting in that it is a molecule that has some elements that are other molecules. But we can get more interesting than that with molecules that have elements that are molecules that have elements that are molecules, etc. Take for example a molecular type that is name a *nucleic acid*.

A nucleic acid has three smaller molecules as its elements: a sugar molecule, a phosphate molecule, and a third molecule that is referred to as a “base”, or as a “nitrogenous base” because it contains nitrogen. But if we look inside of each of these three constituent molecules, we see that they in turn are constructed of other, yet smaller, molecules. Thus, a nucleic acid is a molecule whose elements are molecules whose elements are molecules.

Even more interesting are DNA and RNA molecules, because each of them has as their elements a large number of nucleic acid molecules. (In fact, these nucleic acid molecules come in four different types.) These DNA molecules can become very large – each having billions of clement nucleic acid molecules.

In fact, biochemistry is largely the study of such *macromolecules*, all of which are such compound molecules that whose chemistry consists of many levels of *nested organization* as we have just described.

All of this means that organodynamics requires a mathematical mechanism, or structure, that is complex enough to represent any of these types of macromolecules – while at the same time is simple enough to represent simple molecules that only contain atoms as elements.

### *Implementing Composite State*

In this section, we address the idea of representing certain molecular organizations of a space of atoms as having "molecules nested inside of molecules".

I shall approach this by levering the concept of the *subspace topology* to represent "organizational nesting".

The idea of a subspace topology is that all of the open sets of one topological space are also open sets of also open sets of a "larger" (or same) topological space.

Formally, suppose you have two topological spaces: $\boldsymbol{\tau_A} = (A, T_A)$ and $\boldsymbol{\tau_B} = (B, T_B)$. And suppose further that $B \subseteq A$ and that $T_B \subseteq T_A$. Then $\boldsymbol{\tau_B}$ is said to be a *topological subspace* of $\boldsymbol{\tau_A}$; and $T_B$ is said to be a *subspace topology* of $T_A$.

Notice that all of the open sets in $T_B$ are also open sets in $T_A$. Of course, since $T_B$ is a topology, then B is an open set of $T_B$; which makes B also an open set of $T_A$; which makes B a subset of A. Thus, we do not have to assume that $B \subseteq A$ in the previous paragraph, since we can prove it having been given that $T_B \subseteq T_A$.

It can be seen that topological "subspaceness" is a transitive relationship – just like organizational nesting should be. That is, if topological space $\boldsymbol{\tau_C}$ is a subspace of $\boldsymbol{\tau_B}$, which is a subspace of $\boldsymbol{\tau_A}$, then $\boldsymbol{\tau_C}$ is a subspace of $\boldsymbol{\tau_A}$.

Definition: components: Whenever a $\boldsymbol{\tau_B} = (B, T_B)$ is a topological subspace of $\boldsymbol{\tau_A} = (A, T_A)$, we say that subset B of A is a *component of A*.

This has immediate consequences, because it can be shown that any open set A of any topology on S is a component of S. The reason for this is that for any open set A of S, there is a subspace topology. It is { A, **Φ**}, where **Φ** is the empty set.

The significance of all of this to our MCD exemplar is that it applies to molecules being nested inside of larger molecules. As we have defined the MCD and the definition of component, a nested molecule within an inclosing molecule is a subspace topology, and therefore a *component* of the inclosing molecule.

We are using the idea that a topology of a set provides a "second level of organization of some of the entities associated with S. Specifically, entities that are subsets of S are "converted to" elements of a topology $T_S$ on S. It is this "conversion" that is key to our being able to use both S and $T_S$ together to work multiple "levels of organization" of an underlying space S.

## Third Approximation: Focus on Change of Molecular Organization

Organodynamics is a *dynamical theory*. This means, in the first place, that it is interested in how the *state* of the system being modeled changes over time. In this

section we shall present a mathematical construct that we shall use to model that state change over time, as pertains to the MCD exemplar system that we are using in this article.

Secondly, being a dynamical systems theory also means that the theory accounts for some mechanism of such change. In classical dynamical systems theories, these mechanisms are the *laws of mechanics* and the *equations of motion*. We shall see that organodynamics also provides a mathematical framework for the dynamics of the systems it models. This will also be introduced in this section.

However, we shall not be able to complete our description of the mechanisms of dynamics put forth by organodynamics until the Parts IV and V. In this section, we introduce some of the mechanisms at work in state change within organodynamics.

### *Trajectory*

In this section we shall discuss the form that change-of-system-state over time takes in organodynamics. In other words, we shall define a mathematical construct that we shall use to represent what change has occurred. Later we shall discuss *dynamics*, the mechanism by which that change can occur within the framework of organodynamics.

In classical mechanics, the change of state over time is represented as a *trajectory*, which is sequence of state structures - which happen to be six-dimensional vectors. These six dimensional vectors describe the position and velocity (or momentum) of a particle in classical dynamics. This is appropriate, because classical dynamics is primarily interested in position and velocity (or momentum) of its elements (particles). So much so, in fact, that classical mechanics defines the *state* of its particles to be, precisely such a six-dimensional vector.

However, organodynamics is not primarily interested in the location and velocity (or momentum) of its elements. Rather, organodynamics is primarily interested in the *organization* of the system as a whole. And, like classical mechanics, organodynamics has defined a structure – like the vector in classical mechanics – that represents the *state* its systems. This structure is the *organization of S*, where S is an underlying set of elements. In the previous section, we saw that the mathematical construct used in organodynamics to describe this is an extended topology.

But, as with classical mechanics, organodynamics is interested in the *change of state* of the object of its interest. And, like in classical mechanics, this *change of state* can be represented by a set of *state structures over time*, one for each moment in time being accounted for over some period of time.

For classical dynamics, this set of six-dimensional vectors, one for each time t within a time span. Such a set is called a *trajectory*. The time span of a trajectory can be selected to be continuous; or it can be selected to be discrete – depending upon how the it is desired that time be modeled. If time is modeled as continuous, then the trajectory is represented by a continuous function whose domain is time and whose range is a manifold in six dimensions. If time is selected to be discrete, then the trajectory may be defined as a sequence of *time steps*. Each member of the sequence is a six-dimensional vector that represents the location and momentum of the particle. And its position within the sequence represents an increasing point in time.

However, for organodynamics, this set of state structures is a set of *organizations of S*, or $O_{TRS}$ structures. And, since we are (for now) modeling time discretely in organodynamics, then a sequence of these $O_{TRS}$ structures represents a trajectory.

Therefore, we have the notion of an *organodynamic process*, which we defined as a sequence of *organizations* $O_{TRS}$ *of S* over time, or as a *trajectory of S*. Such a process can be a representation of how our set of atoms in our MCD changes its organization over time.

Recall that, with respect to our MCD example, an $O_{TRS}$ structure represents the current molecular state of our underlying set S of atomic structures. Thus, a *trajectory* on S is a sequence of $O_{TRS}$ structures, or *organizations of S*, each of which represents the molecular organization of S at a specific time point.

### *Organizational Dynamics: Organodynamic Transforms*

So far, we have defined the concept of *trajectory* to represent change of organizational state over time in organodynamics. And, we defined trajectory as a time sequence of $O_{TRS}$ structures, each representing the organizational state of the system at a specific time point. However, we have not yet discussed *system dynamics* in organodynamics – or *how* system state changes from one $O_{TRS}$ structure at one point in time to another $O_{TRS}$ structure at a later point in time. We begin that discussion now. However, we shall continue to embellish the mathematical constructs that we shall use to model dynamics in this theory throughout the remainder of this series of articles. As we introduce increasingly sophisticated exemplar systems into our discussion, we shall at the same time find that we need to add additional mathematical constructs to account for increasingly sophisticated dynamics.

We must first remark that organodynamics is intended to be a *framework* for modeling many specific kinds of complex systems – each of which will have its own peculiar dynamics. So, organodynamics should not dictate a specific set of dynamics (or equations of motion) for all of them. Rather, it should outline a general form, along with a discipline of constraints, which can accommodate the specific dynamics of any exemplar system in the class of dynamical systems for which it seeks to provide a modeling platform[1].

We have a good start with that by defining the concept the way we have for organodynamics. However, we need to take that definition a step further and define a general framework for the dynamics of the time evolution of any exemplar system in our class of interest.

This we shall do with the *system transform*. The general idea of a system transform is that it is a function that maps system state at one point in time to a possibly changed system state at a future point in time. Of course, these system states in organodynamics take the form of an *organization*, which we articulate as and extended topology of the form $O_{TRS}$. Consequently, the domain space of these transforms is the space $\mathbf{O_S}$ of all such organizations on our underlying set S. We can now define:

> Definition: *system transform*: Given an underlying set S of elements, a *system transform* is a function from the space of all possible $O_{TRS}$ structures on S (or $\mathbf{O_S}$) into the space of all possible $O_{TRS}$ structures on S (or $\mathbf{O_S}$).

[1] The reader will note that this situation is not peculiar to organodynamics as a dynamical systems theory. For example, nonlinear dynamics is in the same situation. It does not provide specific equations of motion, as does classical dynamics. Rather, it allows any class of dynamics as long as it presents equations of motion that exhibit "sensitivity to initial conditions", along with some other constraints. Such dynamics typically consists of nonlinear differential equations.

With this definition, we can refine our definition of the concept of *trajectory* in organodynamics. As well as describing *trajectory* as a sequence of $O_{TRS}$ structures, we can say that a *trajectory* on S is as a *composite function of system transforms on the space* $\mathbf{O_S}$ over time.

This perspective on *trajectory* as a *composite transform* provides a mechanism to constrain the reorganization of system state over time from one time point to a later one.

This means that the analog to the *equations of motion* in classical and nonlinear dynamics is the *composite transform on the space* $\mathbf{O_S}$ in organodynamics.

For now, then, we can say that the dynamics of organodynamics incorporates these composite transforms on $\mathbf{O_S}$. However, this is only an approximation to our eventual definition of the mathematical constructs that will ultimately for the definition of dynamics in organodynamics. But for now, lets say that these composite transforms on $\mathbf{O_S}$ are involved in the picture.

### *Operations on Organizations*

In our MCD space, the dynamical mechanisms in organodynamics need to account for chemical reaction that combines, or composes, two or more molecules into a larger molecule. In other words, we need to be able to represent the *composition* of two or more organized entities, *organizations* $O_{TRS}$ *of S*, into a single new entity.

But such a composition is effectively a kind of "fancy" transformation, where we take two *organizations* (molecules) at time $t_1$ and transform the pair to another organization (molecule) at time $t_2$. Of course, such a transform is a binary operation of *organizations*. In general, then, instead of a transform that takes an *organization* of S at one time and transforms it to another organization of S at a second time, we need a binary operation that takes a pair of *organizations* of S at one time and maps them to a single other organization of S at second time.

And, we also want to be able to represent the *splitting* of a single composite molecule at one time into multiple other smaller molecules at a later time. This is *decomposition*. This can be accomplished by defining a second kind of "fancy" transform that maps a single composite *organization of S* to a tuple of organizations of S at a later time.

We need to define a very general kind of transformation that includes the simple transformations that we defined in the previous section, but also includes both the composition and the decomposition operations that we defined in the present section.

The point being made here is that there is a need to provide the mathematics that will enable more complex systems transforms that the one we defined in the previous subsection. Such transforms need to be able to define various n-ary *operations* on the space $\mathbf{O_S}$, rather than the simple unary transformation that we defined above. In fact, these *operations on* $\mathbf{O_S}X\mathbf{O_S}$ need to be developed into an algebraic system, complete with algebraic properties (associativity, distributedness, etc.).

We shall generally refer to these complex operations on $\mathbf{O_S}$ as *organodynamics operators*.

As a dynamical systems theory, organodynamics does not need to specify which specific *organodynamic operators* comprise this set – any more than nonlinear

dynamics should specify which specific nonlinear equations its application should use. However, nonlinear dynamic should provide a general characterization of the class of equations of motion that it intends to deal with. This is namely that its dynamics are concerned with "nonlinear equations that exhibit a sensitivity to initial conditions". In other words, the theory should constrain it range of dominion.

In organodynamics, we have likewise presented a number of qualifying constraints on the dynamics of organodynamics – in particular that it involves composite transforms on $\mathbf{O_S}$. And, in this section, we have extended this notion to some Cartesian product of $\mathbf{O_S}X\mathbf{O_S}X\ldots X\mathbf{O_S}$.

However, organodynamics intends to go further than this and specify a number of *categories* of organodynamic operators – which include various classes of *composition* and *decomposition*. However, such delineation is beyond the scope of the present set of articles. And, we shall leave such considerations to further research.

## Fourth Approximation: Chance and Organizational Change

In this fourth approximation, we are going to add a mechanism to account for chance variation of which *organization of the elements* of underlying set S will obtain (or be selected) as the state of the MCD for a specific time step of an organodynamic process.

### *Nondeterministic Dynamics*

Traditional DSTs, such as classical mechanics and nonlinear dynamics, have *non-random* or *deterministic* dynamics. This means that, given an initial condition, the theories dynamics determines uniquely the state of the system at any time in the process after the initial time.

Deterministic systems typically provide their dynamics in the form of a set of rules, algorithms, procedures, functions or equations that calculate a precise outcome (output) at any future time step. These formulaic dynamics use a set of *initial conditions* as inputs, and calculate unique future state for specified time steps using formulas provided by the system's dynamics.

But organodynamics is intentionally *nondeterministic* – because the complex systems that it seeks to provide a modeling platform for exhibit nondeterministic behavior in their time evolution.

Nondeterministic dynamics work by establishing a set of *constraints* on the future behavior of a dynamical system at any future time. This set of *constraints* has the result of *narrowing* the possible outcomes to a limited number of possibilities. When a set of initial conditions is supplied as input to these nondeterministic constraints, the dynamics eliminate certain possibilities, and thereby narrow the set of possible realized outcomes. However, typically, nondeterministic dynamics do not narrow the possibilities to a single outcome, as deterministic dynamics always do.

In other words, the notion of nondeterministic dynamics has the effect of *generalizing* the idea of *system dynamics* to include merely *constraining* the time evolution of dynamical systems to a limited set of outcomes, rather than requiring that such a time evolution produce *precise* and unique outcomes. Such a generalization, and thus relaxation, enables a dynamical systems theory to attempt to model some very

complex dynamical systems that could not be modeled by requiring a deterministic approach.

Of course, there are many mathematical approaches to implementing a mechanism of nondeterministic constraints. One that comes immediately to mind is the notion of *inequalities of motion*, rather than *equations of motion*.

However, as we shall see next, organodynamics takes a different approach – an approach that leverages probability theory, information theory and the theory of stochastic processes.

## Chance Variation

The particular brand of nondeterministic dynamics employed by organodynamics is called *chance variation*. In these papers, *chance variation* means that a specified procedure, when repeated, always produces a single output (outcome) from a specified set of possibilities. Moreover, which of these possible outcomes is realized in any of these repetitions (or "trials") is subject to *chance*. If the procedure takes input values (*initial conditions*), then chance variation still persists on repeated trials.

In other words, *chance variation* means that the dynamics of the system together with an initial condition merely narrows the set of possible outcomes, rather than specifying a single unique outcome.

Chance variation uses probability spaces (and their probability distributions) to narrow, or constrain, the outcomes. It does this by assigning probability values to each of the possibilities (sample points) in the set of possibilities (the sample space). These dynamics can eliminate certain outcomes from being realized (by assigning them a probability of zero). They can also reduce the likelihood that they will be realized (by assigning them relatively small probability values).

Of course, in organodynamics, these "possible states" are *organizations of S*, $O_{TRS}$ (from $\mathbf{O_S}$), In other words, even though there are multiple possible states $O_{TRS}$ in $\mathbf{O_S}$ that can be realized at time t, given specific initial conditions, the probability distribution (that chance variational dynamics defines) imposes *constraints* on which states can actually be realized at that time. These *stochastic constraints*, then, amount to the nondeterministic dynamics of organodynamics as a dynamical systems theory.

In addition, the initial condition itself need not be precisely known. It too is probabilistic and has its own probability space. Thus, taken together, one or both of the initial condition and/or the process are probabilistic. Taken together, then, the initial condition and the process are subject to chance variation. That is, in organodynamics, chance variation can be involved in both the procedure of the dynamics as well as with the initial conditions.

Therefore, organodynamics will not generally use the mechanisms of classical physics, such as differential equations, to define the systems dynamics. An obvious reason for this is that such approaches are *deterministic*. (Although a stochastic version – stochastic differential equations – is permitted.) Classical dynamics calculates a precise system state as the outcome of a trajectory after time t having been given initial conditions. And this determinism is desirable for classical dynamics; because determinism is desirable for the systems it models. But determinism is *not* desirable for organodynamics, because the class of complex systems that it seeks to model does not generally exhibit determinism.

Another reason that deterministic differential equations, etc. are not use by the dynamics of organodynamics is that its state space is populated by the set of all possible *organizations of S* - each being in the form $O_{TRS}$ – and is not in general a differentiable space. Indeed, it is not in general even a manifold.

Rather, organodynamics should provide its dynamics in some form that identifies a *range of system states* as the outcome of a trajectory after time t – having been given an initial condition (or even a probability space describing the uncertainty involved in knowing the initial condition).

So, organodynamics does not want to be deterministic. Rather, it wants to be more *elastic* than that so as to be able to model a class of systems whose dynamics are more elastic than that. Whereas classical dynamics presents a trajectory as a "path", a more appropriate analogy for organodynamics is that of a "conduit", containing a "bundle" of possible trajectories. The image that comes to mind is a "hose" meandering through "$\mathbf{O_S}$ space" in time that contains several parallel "threads" – each being a possibility, with exactly one of which will eventually be realized at time t.

Thus, the approach of organodynamics to dynamics is to establish a set of criteria that constrain which *organizations of S* will be permitted to be "selectable" as a realized state for certain time points within an organodynamic process. And these constraints are probabilistically articulated.

## Stochastic Constraints

Probability theory offers two general mechanisms that provide some constraint on the behavior of an "experiment", or chance-influenced process, over time. These are

1. Outcome weighting through probability assignments
2. Outcome dependency

We shall describe these in more detail, and how they shall be employed in organodynamics, in the next two subsections.

## Probability Assignments

Each *organization* $O_{TRS}$ in the set of all possible organizations of S $\mathbf{O_S}$ shall be assigned a probability value. This probability represents the likelihood that the *organization of S,* $O_{TRS}$, to which it is assigned is to be realized for a given time point.

Making this assignment implies many things. One is that we are now treating $\mathbf{O_S}$, the set of all possible organizations of S, as a *sample space* within a *probability space* whose *sample points* are the members, $O_{TRS}$, of $\mathbf{O_S}$.

Moreover, taken together, these sample points and their assigned probabilities form a *categorical probability distribution*.

Note that we *cannot* say that this probability distribution is a *probability mass function* (PMF), because our sample space is not comprised of *numbers*. Rather, it is comprised of complex constructs of the form $O_{TRS}$, each of which is an *extended topology*. If there were some meaningful way to associate some real number with each $O_{TRS}$, then we could substitute those real numbers for each $O_{TRS}$. This would permit the use of a probability mass function, rather than a categorical distribution.

The disadvantage to the categorical distribution over the probability mass function is that ordinary statistics, such as the mean and standard deviation, cannot be calculated for the categorical distribution – because the sample points have not numerical values! We shall discuss the ramifications of this fact in some depth later in this series of articles. Needless to say, they are profound! In fact, they dictate that organodynamics must use information theory rather than mathematical statistics as its mathematical foundation.

In any event, we do have a sample space ($\mathbf{O_S}$). And we can assign probabilities to the members of that sample space (each $O_{TRS}$). And these are two of the three requirements to have a probability space. The third requirement is a set of *events* comprised of certain "logical combinations" of the sample points. Once the set of events has been developed, the probabilities that have been assigned to the sample points can be used to deduce probabilities for the events as well. This set of events is defined by the "combination rules" known as a *sigma-algebra*.

In other words, we can create a probability space for any organodynamic system by using the sample space $\mathbf{O_S}$ to develop a sigma-algebra of *events*. We can then assign (somehow) probabilities to the sample points, and use these to calculate probabilists for the events. These are the three ingredients necessary to define a *probability space* for organodynamics.

In the previous approximation 3, we advanced to the point where we had developed the concept of a system process – meaning a sequence of *organizations* of S, $O_{TRS}$, each representing a time point. Such a structure is used to model the way that a dynamical system changes its organization over time.

What we want to do in approximation 4 is to introduce chance variation into each time step of a system process. This will enable us to say that, at any time step t in the process, there is some probability that a certain organization of S, $O_{TRS}$, will be realized at time t within this process.

This we shall do by starting with the *system process*, and at each of its time steps, we replace the $O_{TRS}$ there with a probability space whose sample space is $\mathbf{O_S}$. This new process is, by definition, a *stochastic process* whose sample spaces are an $\mathbf{O_S}$; that is, whose sample spaces for each time step t are *organizations,* $O_{TRS}$, of S. We shall name such a stochastic process an *organodynamic process*. We shall develop these ideas more formally below.

An *organodynamics process* then depicts the time evolution of one of our target exemplar dynamical systems. It does so by using a probability space (or sequence of them) to impose certain chance variational constraints on the outcomes of the process. In so doing, this organodynamic process prescribes a mechanism for the dynamics of organodynamic systems. These are generally nondeterministic, or stochastic, dynamics. And they track organizational change-of-state.

The development of the *organodynamic process* will constitute the fourth approximation of organodynamics. We develop that for the MCD example below.

## Stochastic Dependency

Even stricter chance variational constraints can be imposed on these stochastic dynamics by the introduction of dependent stochastic processes. The idea is to use

conditional probability to refine the probabilities associated with each time step in the process by taking into account the outcome of the previous time step.

We have previously discussed that conditional probability imposes a constraint on a stochastic process. The constraint is one of reduced uncertainty. It can be shown that a conditional distribution has more certainty (less uncertainty) than it corresponding unconditioned distribution as long as there is stochastic dependence. This is expressed by the following inequality from information theory: $H(X|Y) \leq H(X)$. We shall develop these ideas more formally in Parts IV and V.

But this fact, and many of its implications, provide a set of constraints on the growth of uncertainty in a stochastic process, and can provide a foundation for the dynamical mechanism that we shall use to drive system dynamics of our theory of organodynamics. It further turns out that *information theory* provides a powerful compendium of mathematical constructs from which we shall develop these dynamics. We shall develop these ideas more formally in Parts IV and V, where we shall see that they depend upon the use of condition distributions and dependent stochastic processes.

This type of stochastic process is often seen in applied probability. For example, the Markov processes are an example of this. A Markov process is a stochastic process that used, at every time step, conditional probability distributions that are conditioned on the realized outcome of the previous time step. In fact, a Markov process utilizes exactly the same conditional probability distribution for each time step in the sequence. That is, Markov processes are time-homogeneous.

The development of the *dependent organodynamic processes* will constitute the fifth approximation of organodynamics. We shall develop that for the MCD example in the following section of the current article.

### *Support for Stochastic Organizational Change*

To this point we have discussed a number of structures that build upon each other and that organodynamics uses to model the chance nature of organizational change over time in complex dynamical systems. These include 1) an *underlying system S* of arbitrary elements, and 2) a set $\mathbf{O_S}$ of *organizations* $O_{TRS}$ of S.

In this section, I shall expand these constructs into a number of other constructs from probability theory that I shall customize specifically for the purposes of organodynamics. I will then exemplify these constructs by examples from MCD.

These constructs were introduced in Part I. It is time now in Part II to give them a more formal treatment by describing them in terms of the definition of organizational system state in organodynamics – the extended topologies $O_{TRS}$, and their *state space* $\mathbf{O_S}$.

In addition, I shall address what it takes to approach an application that one desires to model using organodynamics – such as our MCD exemplar system – and develop and organodynamic model of it.

In this section, we shall more formally develop a number of new mathematical constructs from this state space $\mathbf{O_S}$ to give rise to the stochastic aspects of organodynamic theory. This is made possible by associating probabilities to the states $O_{TRS}$ in $\mathbf{O_S}$, thus promoting $\mathbf{O_S}$ to a sample space and subsequently to a probability space. From that, we develop a number of dynamical constructs in the form of

stochastic processes on those probability spaces. Thus, a mathematical framework for the dynamical aspects of organodynamics is provided.

It should be understood that these constructs form the basics of organodynamics theory.

## Underlying System

A complex system being modeled with organodynamics initially consists of a set S of *elements*. As a modeling platform, organodynamics is interested in how these elements can be grouped, and how the elements within those groups are interrelated to each other. Collectively, these interests are referred to as *organization*.

Formally, an *organization of S* is a collection of subsets of S that form a topology on S. This topology on S, or *organization of S*, is the principle unit of organization used by organodynamics.

When modeling a system with organodynamics, it is important to identify some entity within the target system that will be the modeling analog of the elements of S. The set of all such entities within the target system will be the analog of the set S. In the MCD exemplar system, the atoms (atomic structures) are the analogs of the elements of S, and the closed space of atoms is the analog to the set S.

Additionally, the modeler must be able to find some collection-type entity within the target system that is a collection of the element entities. In the MCD example, these are the molecules. In addition, a collection of these collection entities must be able to include all of the element entities – or be enhanced to do so. In the MCD system, the molecules collectively contain – or "cover" – most of the elements (atoms) in the space. If there were some "free atoms" in the space, then they would not be covered, of course. But I was able to take each such "free atom" and define a set that contained only its element – a singleton set. By adding these new sets to the collection of "molecule" sets, we have a set of collections of atoms that, collectively, "covers" all of the atoms in the space.

So, this is the basics of what is needed in order to model a target system using an organodynamic model: Some entity in the target system (such as the atoms) must be identified that can be considered as the elements of the underling set S. Some collective entity (such as molecules) must be found within the space such that instances of it can be considered to be subsets of S, because each of them has elements that are also elements of S. Moreover, the set of all of these subsets of S must "cover" S, or be extendable to cover S. Once these entities can be identified within the target system, then the target system is a candidate for being modeled by organodynamics.

The next step in modeling the target system organodynamically is to use this "cover" of S to create a topology on S. Taking all unions and intersections of the subsets in the cover and adding the empty set if necessary does this. This action results in a *topology* on S, and is the basis for the structure that we have symbolized as $O_{TRS}$, the structure that represents system state in organodynamics.

The complete construction of an $O_{TRS}$ requires an additional step, as we defined it above. This step adds to this topology in a manner that captures the interrelationships among the elements (e.g. atoms). However, we shall not reiterate that here.

In summary, starting with an "underlying set", these are the basic entities that must be identified with a target "complex system to be modeled" in order to be able to use organodynamic theory to construct a model of the target system.

## Organodynamic State

Since organodynamics is primarily interested in *system organization*, then it defines a structure that represents the organization of system S at some specific time t. This is similar to classical mechanics, in that the theory identities its object-of-interest (the particle), and then defines a mathematical structure that represents that particle a specific time – a six-dimensional vector representing location and velocity of the particle.

Organodynamics, on the other hand, is interested in the *organization* of the entire *system*, at any point in time (not anything specific about the individual *elements* of S). This being the case, then when organodynamics defines the notion of *state*, it must define a structure that describes how all of the elements of S are collectively organized. We have seen that this structure is named $O_{TRS}$, and that it consists of a number of substructures that describe how the elements of S are apportioned to various sub-groupings. $O_{TRS}$ also describes the interrelationships among the elements within those groupings. Organodynamics defines $O_{TRS}$ as an extended topology on S.

In MCD, the $O_{TRS}$ structure describes how the atoms of S are allotted to the molecules that are currently configuring S. Such a structure is capable of describing how the atoms of S are apportioned across its molecules and sub-molecules, as well as describing the interrelationships (chemical bonds) among the atoms of a molecule of sub-molecule.

## Organodynamic State Space

An *organodynamic state space* is the set of all logically possible *states* of the form $O_{TRS}$ for an underlying set S. This set is referred to by the symbol $\mathbf{O_S}$. Notice that at any one point in time, exactly one of these $O_{TRS}$ structures in $\mathbf{O_S}$ defines the current state of S. That is, this *state space* can be understood as a set of alternative states.

In MCD, then, $\mathbf{O_S}$ is the set of all logically possible ways that an underlying set of atoms can be organized into molecules.

## Organodynamic Sample Space (OSS)

An organodynamics sample space is simply an organodynamic state space. However, this appellation is applied in the context that, as a state space, it will be out to use as the sample point of a probability space. In other words, once we have decided to assign probabilities to each of the states $O_{TRS}$ within an organodynamic state space, then that state space has become a *sample space* within a probability environment. We shall continue to refer to this sample space as $\mathbf{O_S}$, even within its new role as a sample space – of some probability space that we are about to define below.

In a particular MCD environment, with its particular mix of underlying atoms, each specific organization of the atoms into a molecular configuration $O_{TRS}$ has some probability. Such probabilities can either be empirically observed, or they can be theoretically derived from various assumptions about the particular MCD environment.

Admittedly, the assignment of individual probability values to individual $O_{TRS}$ sample points would be an intractable proposition due to their massive number sin most complex systems. The only approaches left must be based upon theoretical consideration, perhaps combined with empirical verification. Statistical mechanics has exactly this same problem. There approach has been a combination of the theoretical followed by empirical verification.

Statistical mechanics has good physical reasons to theoretically assume a uniform distribution. And, they can provide some empirical verification through the application of Bayesian analysis [Jaynes 1957]. While the technical considerations of probability assignments are beyond the scope of these articles, let me say there are fruitful approaches to combining both the theoretical and empirical using information theory (relative entropy) and Bayesian analysis in order to develop a regime for assigning probabilities.

Once we have made such probability assignments to the states $O_{TRS}$ in $\mathbf{O_S}$, then we effectively have a *categorical probability distribution*. Thus, at this point, $\mathbf{O_S}$ is operating effectively as a *sample space*.

## Organodynamic Probability Space (OPS)

Once we have 1) a sample space $\mathbf{O_S}$, and 2) probabilities for all of the sample points (states) in $\mathbf{O_S}$, then we have the principle ingredients of a *probability space* on $\mathbf{O_S}$.

A *probability space* [Ash, et. al. 2000] is a complete formal system that establishes all that is necessary in order to work with probabilities on some system of elements. Suffice it to say that a probability space is a formal system that consists of three parts:

1. $\Omega$: A set of elements, $x_i$, treated as sample points
2. **F**: A sigma-algebra on those sample points
3. $\rho$: Probability assignments on the sample points[2]

We shall now describe each of these in the context of a finite set of elements – which is the case dealt with (for now) by organodynamics. The case for infinite elements is well defined, but has an extended description and will not be dealt with here.

The set of elements, $\Omega$, is a set of alternative outcomes of an "experimental trial", exactly one of which will be realized at the end of the trial. In organodynamics, $\mu$ is $\mathbf{O_S}$, our set of all possible *organizations* $O_{TRS}$ of our underlying set S. That is, we ultimately want to assign a probability to each possible way $O_{TRS}$ that our set of atoms S can be organized. Therefore, we want to turn $\mathbf{O_S}$ into a probability space.

The essential purpose of the probability space, then, is to construct a regime that *measures the likelihood* of each of the elements in the space being the one that is actually realized at the end of the trial – where exactly one of them will be. In the case of organodynamics, we are, at this point, shifting our focus away from the underlying space of elements S, and toward our space whose "elements" are $O_{TRS}$ - *organizations of S*.

[2] For continuous probability spaces, the probability assignments are made to the sigma-algebra of events, rather than the sample points. However, for discrete or finite probability spaces – which we are always dealing with in organodynamics – the probability assignments are made to the sample points, and then derived for the events in the sigma-algebra.

Lets next discuss item 3 of the above list, the set $\rho$ is the probability assignments to each of the elements in $\Omega$. In organodynamics, these are the probabilities that specific *organizations of S*, as articulated by the constructs $O_{TRS}$, will describe the way that the organodynamic system is organized at some time step.

We shall symbolize this probability distribution as p($\mathbf{O_S}$). By convention of course, the probabilities are non-negative and sum to one.

Finally, item 2 is the set of *events* that can be contrived from combining multiple sample points from $\Omega$ using logical operators. The logical operations that are permitted (in a probability space) are established by the definition of an algebraic structure called a sigma-algebra. A sigma-algebra, F, is a set of subsets $X_k$ of $\Omega$ that are considered proper events that can occur within the space.

To insure that F is mathematically viable and interesting, it must exhibit certain properties that qualify it as a sigma-algebra. These are 1) $\mu$ is an element of $\Omega$, 2) $\Omega$ is closed under set-complementation, and 3) $\Omega$ is closed under countable unions.

In MCD, then, we are now focusing on making $\mathbf{O_S}$ into a probability space. This means that $\mathbf{O_S}$ is $\Omega$ in probability space parlance. This also means that our sigma-algebra F will consist of subsets of $\mathbf{O_S}$. Thus, our sigma-algebra $\Omega$ contains sets of *organization of S*, $O_{TRS}$. But each $O_{TRS}$ in MCD is a way that the atoms of S can be organized (composed) into molecules. Thus, each subset in $\Omega$ is a collection of ways that S can be organized into molecules.

## Organodynamic Probability Distribution (OPD)

A discrete "probability distribution" is essentially an abbreviation of a discrete probability space that articulates the minimal information concerning the space. For a finite probability space – such as we are dealing with in organodynamics – that information includes a specification of the sample points of the space, together with the probability assignments to that sample space. The sigma-algebra that defines the allowable events is unspecified, for brevity sake.

Probability theory and mathematical statistics define a number of types of "probability distributions". However, of those, the kind that applies to organodynamics is called a *categorical probability distribution*.

Most of the other kinds of "probability distribution" require that there be some numerical function that has also been defined on the sample space. Such a function maps each sample point (which is generally on-numeric) to some numerical value. We shall refer to such a function as a *value function*. However, the sample space used by organodynamics, the $\mathbf{O_S}$ whose sample points are topologies $O_{TRS}$, has no meaningful mapping to the real numbers. That is, there is no semantically useful *value function* in organodynamics. As a result, the only type of probability distribution that is semantically useful in organodynamics is the *categorical distribution*.

I shall leverage this idea of to specify a particular kind of probability distribution used by organodynamics. I shall then define the

> *organodynamic probability distribution* (OPD) as a categorical distribution whose sample space is an $\mathbf{O_S}$.

Consequently, an OPD can be articulated as

OPD = { ($O_{TRSi}$, p($O_{TRSi}$) | $O_{TRSi}$ $\mathbf{O_s}$, for some organodynamic sample space $\mathbf{O_s}$.}

## System Process

We have already defined (discrete) *system process* above as a sequence of *system states*, one for each time step in the discrete process. Since system state in organodynamics takes the form $O_{TRS}$, then an organodynamics system process is a sequence of $O_{TRS}$ structures, one for each time step in the process.

An organodynamics system process, then, represents the time evolution of an organodynamic system when the outcome (specific *organization* $O_{TRS}$) is known at each time step.

However, organodynamics is a stochastic theory in which the outcome of any time step is not precisely known and cannot generally be precisely determined by the dynamics of organodynamics.

We mention *system process* at this time, because we are going to build upon the concept in the next subsection to construct a kind of mathematical process that does suffice to represent the stochastic time evolution of a dynamical system that is modeled by organodynamics.

## Organodynamic Stochastic Process (OSP)

The dynamics of organodynamics intentionally does not "know enough" to be able to precisely determine in advance the outcome of any time step of the time evolution of the complex dynamical systems that it models. This is because the systems that organodynamics seeks to model are generally nondeterministic.

What is known (in advance of a time step) is a *probability space* that represents all that is known about that time step. This knowledge includes the possible outcomes of that time step (its finite sample space), and probabilities of those sample points.

The dynamics of organodynamics cannot precisely predict the realized outcome of any of those time steps. But we want those dynamics to use what it does know about the time step (its probability space) to say as much as it can about the eventual realized outcome. What it can do, at the point of this fourth approximation, is to take into account the probabilities of these sample points in order to *constrain*, and to *narrow*, the potential outcomes.

Therefore, each time step involved in this time evolution will be represented by the probability distribution of the probability space associated with the time step. This means that we can represent such a process – and take chance variation into account – by articulating the time evolution of the system as a sequence of probability distributions.

Therefore, in order to develop a mathematical process structure for modeling the time evolution of systems, we shall begin with the *system process*. However, for the *system organization* $O_{TRS}$ at each time step, we shall substitute the probability distribution of the space $\mathbf{O_s}$ at that time step. This probability distribution, as we just defined it, is p($\mathbf{O_s}$).

Thus, we now define the *organodynamics stochastic process* (OSP) to be a sequence of probability distributions $p_i(\mathbf{O_s})$, one for each time step, i, in the time evolution of the dynamical system being modeled.

Notice that this definition permits the organodynamics probability distribution $p_i(\mathbf{O_s})$ to change at every time step. Generally we do not expect this to happen. Typically, we expect organodynamic systems to exhibit the same $p_i(\mathbf{O_s})$ for some finite number of contiguous time steps – all "under the sway of" the same probability distribution. Subsequently, at some particular time step, the process will typically change probability distributions $p_i(\mathbf{O_s})$ and repeat for some other finite number of time steps. This will typically continue for some countable number of repetitions. We call this behavior, *piecewise homogeneous dynamics*. Notice that *homogeneous dynamics,* in which only one $p_i(\mathbf{O_s})$ is used throughout, is a special case of *piecewise homogeneous dynamics* – as is the case when the probability distribution changes every time step.

The *organodynamics stochastic process* (OSP) will henceforth be the used as the principle mathematical construct for modeling the time evolution of dynamical systems in organodynamics.

However, in the next section we shall make one more enhancement to the OSP before it takes its final form.

## Fifth Approximation: Stochastic Dependency

It often occurs within dynamical systems that knowing the outcome of one time step changes ones notion of what the probabilities are for the next or later time step. For example, in a game of poker, knowing which cards have already been dealt changes the probability distribution for the possible card hands thereafter in the game.

This situation is called *stochastic dependency*, or *statistical dependency*, because the probabilities values of a future time step *depend upon* the outcomes (what cards have already been played) of the present and past.

That is, knowing the past can change the probability distribution of the future. Whenever that occurs, we have *stochastic dependence* (or st*atistical dependence*).

As we shall discuss at length throughout the remainder of this series of articles, stochastic dependence is responsible for the "taming" of stochastic processes and constraining them from going "wildly out of control" as time progresses. In fact, it is the probabilistic mechanism of stochastic dependence, implemented by conditional probability, from which the mathematical dynamics of organodynamics derives.

Consequently, this fifth approximation, which introduces stochastic dependence into organodynamics, is essential for the mathematics of organodynamics. All of the mathematics presented in the earlier approximations above was simply prolog – providing the essential mathematical preliminaries in order to be able to introduce stochastic dependence into organodynamics.

In what remains of these five articles, we shall be developing somewhat elaborate mathematical constructs to exploit these ideas and formalize them as the theory and practice of organodynamics.

To discuss this mathematically, we need to define a new kind of probability distribution. This is a distribution that assigns probabilities to the sample space based upon its

knowledge of previous outcomes. This kind of distribution is called a *conditional probability distribution* (symbolized "p(Y|X)"). Of course we still have to initial probability distribution that *does not* take into account any previous outcomes. We shall call this the *unconditioned probability distribution* (symbolized "p(Y)").

If the *unconditioned probability distribution and the conditional probability distribution* are different – have different probability assignments (that is, $p(Y|X) \neq p(Y)$), then the probability values *do depend on* the earlier outcomes. In this case, we have stochastic dependence. In fact, the later time steps *depend upon* the earlier ones. However, if the conditional distribution is the same as the initial ones (that is, $p(Y|X) = p(Y)$), then we have *stochastic independence*.

But there is more…

If there is stochastic dependence, then it can be shown that the *conditional distribution* has more certainty about it than does the initial distribution! That is, if there is stochastic dependence, then the revised distribution (the conditional one) is more deterministic – provides tighter constraints – than the initial distribution (that is, $p(Y|X) < p(Y)$).

This is why "card counting" (conditional probability) in poker is more deterministic – provides "more information" – than not card counting (the initial distribution).

It is an elementary theorem in information theory that conditional probabilities are never less certain (never have higher entropy) than do their "unconditional" counterpart distributions (that is, $p(Y|X) \leq p(Y)$).

This is why the use of conditional (or dependent) stochastic processes abounds in applied probability applications. For example, Markov processes are one of the simplest types of dependent stochastic processes, and they are heavily used in applied probability.

To introduce stochastic dependency into organodynamics, one needs to modify the initial probability distributions for each time step of the OSPs in approximation 5 above so that they become conditional probability distributions (as in Markov chains).

This act permits one to apply the peculiarities of one's application and insert them into the organodynamic model being constructed. In the MCD exemplar, for example, suppose one know how the atoms of S are currently organized into molecules. This means that one know the organizational description, $O_{TRS}$, of the MCD space. From this particular organization $O_{TRS}$, it is possible for the space to next directly reorganize to some other organizations, and impossible to directly reorganize to other organizations. Then the conditional probability of directly reorganizing to those others is zero. And, knowing the chemistry and counts of the particular atoms involved, it is more or less likely to directly change from the present organization $O_{TRS}$ to the others – each having its own conditional probability.

Conditional probability is also away to account for the effects of system nesting. The essential consequence of system nesting is the inaccessibility of deeply nested elements to other elements that are external to its nesting hierarchy. In fact, the essential nature of system nesting is to implement constrained accessibility. This fact effectively eliminates certain deeply nested organizations from being the next organization to which a current organization can directly transition. In other words, system nesting can be accounted for by the proper assignment of conditional probabilities.

In other words, the nature of the application itself dictates these conditional probabilities, and the conditional probability distribution. And, this conditional probability distribution describes a set of constraints (conditions) that model the dynamics of the MCD system as a whole. Moreover, this particular conditional probability distribution works for all time steps in the MCD dynamical system as long as that system remains chemically closed – no atoms or energy is allowed in or out.

So, by determining the conditional probability distribution for the space $\mathbf{O_S}$, the dynamics of a particular MCD space (with a particular atomic mix and amount of energy) can be modeled.

We have just alluded to the concept of a conditional probability distribution. In the case that the sample space of such a distribution is the space $\mathbf{O_S}$, then we shall refer to the distribution as an *organodynamic conditional probability distribution*.

In conclusion of our description of this fifth approximation of the theory of organodynamics, we redefine the *organodynamics stochastic process* (OSP) to be:

> Organodynamic Dependent Stochastic Process (ODSP): a sequence of organodynamic conditional probability distributions, one for each time point of interest.

This is our final refinement to the definition of the mathematical structure that we are calling an *organodynamics deterministic stochastic process* (ODSP). As indicated, the ODSP will be the principle construct in organodynamics for representing time evolution in complex dynamical systems.

The other constructs, OSS, OPS, OPD, ODPD and OSP were simply foundational ideas used to build up the necessary machinery to enable the definition of the ODSP. It is the ODSP that define the mathematical dynamics within organodynamic theory.

Whereas, an organodynamic stochastic process (OSP) is a *sequence of organodynamic probability distributions* (OPDS), an ODSP is more elaborate. Rather, it is a sequence of *conditional probability distributions* in the form of *organodynamic dependent probability distributions* (ODPDS) – one for each time step in the process. Each of these conditional distributions contains all of the conditional probabilities for the present time step, conditioned on the outcomes of the past. These conditional probabilities inculcate any stochastic dependencies between the past time steps and the current time step.

In the simplest case, the stochastic dependencies only go back one time step into the past. This simplifying condition is called the *Markov condition*. This occurs whenever considering dependencies that go further back in the past than the previous time step gives one the same conditional probabilities as only going back one time step. This situation is also referred to as “memorylessness”.

In other words, “Markov condition”, is expressed mathematically by saying that “the probability of the current time step conditioned on all past time steps is equal to the probability of the current time step conditioned on the previous time step only:

$$p(x_{n+1} \mid x_n, , x_{n-1},\ldots, x_1) = p(x_{n+1} \mid x_n)$$

In organodynamics, we shall generally assume the Markov condition to hold, if only because it markedly improves the tractability of organodynamic models. Making such an assumption is usual practice within stochastic process applications. Without this

assumption, the conditional distribution (ODPD) at each time step must be conditioned on all past time steps.

In practice, this means mathematically, that a Markov transition matrix represents each time step in an ODPD. In other word, the dynamics of complex adaptive system is represented in an organodynamic model represented by a sequence of Markov transition matrices, one for each time step of the process. In the time-homogeneous case, the same Markov matrix represents all time steps. In that case, as with any Markov chain, the single Markov matrix captures the entire process.

In organodynamics, however, we typically expect the process to be *piecewise homogeneous*. This means that a sequence of Markov transition matrices must be used – one for each change in ODPD throughout the time series that is the ODSP.

In organodynamics, of course, the sample points are extended topologies of the form $O_{TRS}$. Thus, a Markov process whose sample space consists of organodynamic *organizations* of the form $O_{TRS}$ is a special case of an ODSP called an *organodynamics Markov process*, or OMP. If the OMP is a Markov chain (is time-homogeneous), then it is called an *organodynamics Markov chain*, or OMC.

## Sixth Approximation: Stochastic Regulation

We have indicated how the application of the concept of conditional probability, and its offshoots such as dependent stochastic processes, provide a regime for constraining the behavior of stochastic dynamical systems over time.

The mathematical treatment of these issues is the product of *information theory* – which formalizes these ideas in the form of a set of *entropic functionals*, which effectively describe the degree to which stochastic dependency in a stochastic process does or does not implement various degrees of constraint over time. In fact, information theory provides a rich characterization of these constraints, and forms the foundations for the dynamics of organodynamics.

These so-called *entropic functionals* include entropy, joint entropy, conditional entropy, relative entropy, mutual information and entropy rate. Collectively, they provide a formidable arsenal of equipment to model any number of application issues, including uncertainty, predictability, stability, opportunity, risk and other considerations or primary interest to modelers of stochastic dynamical systems.

The issues just mentioned all pertain to the degree to which a dynamical system exhibits *regular behavior*. These issues require an investigation of the conditions under which “regular behavior” can be expected over time, and when it cannot. Also at issue is the categorization of various kinds of behavior in dynamical systems, whether regular or not. For example, is the behavior asymptotic? Is it unbounded? Is it periodic? Is it “irrational”?

All of these issues pertain to the characterization of the dynamics of a stochastic system. And organodynamics will exploit these entropic functional tools from information theory to provide mathematical characterizations of these system dynamics.

We must delay any analytical or formal considerations of these issues until Parts IV and V of this series of articles, where we leverage a considerable amount of intellectual machinery from information theory to extend organodynamics in order to hope to present a comprehensive theory of highly complex adaptive dynamical systems –

systems whose degree of complexity obviate their being faithfully modeled by extant dynamical systems theories.

## Conclusions

In this second article in this series on Organodynamics, we have used a particular exemplar application, *molecular compositional dynamics* (MCD) to help us subject our initial view of organodynamic theory, presented in the first article, to further scrutiny.

We have used this scrutiny to add quite a bit of flesh to the bones of this theory. So that, at this point, our theory is beginning to have some specificity that it lacked at the end of the first article – an article that presented an overview of organodynamics.

All in all, the ideas presented in the first article held up pretty well. More often than not, we ended up refining and embellishing those ideas, rather than having to replace any of them.

## Preview of Part III

In the next article, we shall subject our current definition of organodynamics as a mathematical framework for modeling highly complex dynamical systems to yet another exemplar system – one that is considerably more complex than our "toy" MCD exemplar. This time, the exemplar will be *human cognition*, as represented through functional magnetic resonance imaging (fMRI). This should present a stronger challenge to the theory we are developing.

In addition, we shall also look at the arsenal of mathematical constructs that are offered by information theory, in the form if *entropic functionals*, to see what kind of value they can offer in terms of providing a descriptive dynamics for the probabilistic processes that we have so far defined as the principle modeling mechanism in the framework that is organodynamics – primarily featuring the *organodynamic dependent stochastic process* (ODSP).

This mathematical arsenal includes a number of entropic functions. For example, we shall find *relative entropy*, *mutual information* and *entropy rate* to be particularly applicable and useful to organodynamics modeling.

## References

[Ash and Doleans-Dade 2000] Ash, Robert B., Catherine A. Doleans-Dade; *Probability and Measure Theory*; Second Edition; Academic Press; San Diego, CA; 2000. Books; New York, NY, 2011.

[Jaynes 1957] Jaynes, E. T.; *Information Theory and Statistical* Mechanics; *The* Physical Review, Vol. 106, No 4, 620-630, May 15, 1957. Also at http://www.weizmann.ac.il/complex/tlusty/courses/InfoInBio/Papers/JaynesInformationTheory.pdf, 1957.