Organodynamics: A General Theory of Dynamical Systems based on Chance Organization

Part III of V: Further Analysis

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Modeling Cognition - A Neurological Exemplar System

In Part I of this series of five articles, that introduce organodynamics as a new dynamical systems theory, I presented the overall structure of the new theory. In the second article, Part II, I develop the theory at a high level of organization by analyzing an exemplar application – that I shall call "molecular compositional dynamics", or MCD.

In the present article, Part II, we shall subject the theory of organodynamics to further scrutiny by attempting to use it to develop a dynamical model of a second exemplar system – that of human cognition. The model is developed using brain data obtained though the use of *functional magnetic resonance imaging*, or fRMI.

Nicolas B. Turk-Browne reviews research into this area of neurology and brain science in an article published in the November 2013 issue of Science [Turk-Browne 2013]. The article opens with a provocative question-and-answer retort:

> Why does the brain, and not the pancreas or any other human organ, arouse such popular interest? The key reason is that the brain implements the mind.

The analogy that seems to be suggested here is that the brain is a "platform" like computer hardware and the mind is like computer software that runs on it.

Given this relationship, Turk-Browne asks the discriminating question:

> What is the best way to find the mind in brain data?

Turk-Browne points out that contemporary neurological research is turning up a good deal of brain data by the use of fRMI. So, he is posing this question in the context of this kind of research activity.

Brain activity is observed using fRMI by detecting "neuronal activity" in the brain. These interactions between neurons essentially constitute "brain function", or "cognition". It is the dynamical behavior of these interactions, then, that constitutes "brain function", or "cognition", or "mind". Thus, by detecting and mapping these sequences of interactions (communications) among neurons that empirical observation can discern the distinction between brain and mind, and can therefore identify cognitive processes, or "cognition" (mind).

My interpretation is that this relationship between brain and mind is analogous to discerning "software" in a computer by observing the sequence of hardware instructions that get executed over some time period in a CPU (or GPU).

Now, brain science has already succeeded at organizing the brain into *regions*, which have been traditionally interpreted in neurology as having homogeneous and discrete function. Looking for homologies between brain region and mental function can provide a test for this supposed independence of brain regions.

Such a test would involve having subjects perform well-defined "active tasks", such as either maintaining attention or switching attention. fRMI is used to observe the subject's neuronal activity during those tasks. The data obtained would describe the neuronal interactions (mental activity) of the subject while performing such tasks. One item of interest is whether (and the extent to which) this interneuron communication is confined within the same brain region – or whether the communicating neurons lie across brain regions. Such observations could reveal, "how mental activity (thought) is organized", as opposed to "how the brain is organized". Of course, such observations can also imply how the various brain regions participate in the organization of mental activity.

In any event, this kind of fRMI measurement can reveal the areas of the brain (collections of neurons) that intercommunicate in the process of performing defined task – and in so doing assist in the understanding how the mind (cognition) is organized – and what is the relationship between the organization of the mind and that of the brain.

It appears to this author that such a study is a "poster child" modeling application of organodynamics. If so, then organodynamics can be used to "uncover the fundamental principles of cognition and behavior" - to quote Turk-Browne, at the end of the first paragraph of his article, as a worthwhile goal [Turk-Browne 2013].

Whole-Brain Functional Connectivity During Cognitive Tasks

The fRMI research reviewed by Turk-Browne confirmed that the "restful state" (the "default cognitive activity"), defines a particular "functional structure" of the brain. Essentially this "default mental activity" described how the various regions of the brain interact when the mind is resting. Obviously, this "resting activity" was described here by enumerating the regions of the brain that were organized via inter-neuronal interaction, and how that organization changed over time during "resting mental activity" to constitute a specific kind of mental activity – that is "resting".

Mental Activity in the Brain

Since a sequence of organizations of the brain region was used to describe this "default behavior", the study refers to it as the "default network". However, the study further revealed that this "default network" is not "stable". That is, "Over time, its constituents interact differently with each other and with the rest of the brain." [Turk-Browne 2013, p. 581].

In other words, even when the brain is engaged in the same mental activity ("resting"), over time the way that the neurons of the regions of the brain interact is not stable – that is, changes over time. So the *organization* of the neuronal interactions over time (the organization of cognition) changes over time – even for the same mental activity.

However, to more thoroughly study cognition, other well-defined mental activities (other than "rest") should also be studied. Says Turk-Browne, "Studying connectivity during (non-resting) tasks is a more direct way to understand how cognitive processes are realized in the brain."

Organodynamic Interpretation

In order to construct an organodynamic model of this fRMI domain of application, we must identify entities of this domain that we can reasonably map to the mechanisms of organodynamics, including: basic elements, an underlying set S of these elements, a way to organized these elements of S into structures of the form O_{TRS}, and the collection of all of these O_{TRS} into a state space $\mathbf{O_S}$. If we cannot find such an association, then we must conclude that organodynamics is not a suitable theory for modeling cognition in the human brain.

If we can make these associations, then, we must proceed to develop the dynamical constructs of organodynamics from these static ones. These include interpreting the state space $\mathbf{O_S}$ as an organodynamic sample space (OSS) and then defining from it an *organodynamic probability space* (OSS) and probability distribution (OPD). With this equipment, we can then finally develop our organodynamic model of the time evolution of cognition (or, "thought") in the form of an *organodynamic stochastic process* (OSP).

A final step in developing an organodynamics model of human cognition based upon the research of [Turk-Browne 2013] is promote the OSP mentioned above to an organodynamic dependent stochastic process, or ODSP, as described in Part II of the present series of articles.

In addition, we shall go beyond the mechanisms that we provided in the previous article regarding dynamics. Here we shall introduce the application of information theory concepts to further characterize the stochastic dynamics of organodynamics. We shall introduce the use of several entropic functionals and show how they measure constraints in behavior of stochastic systems over time. These functionals are all based on statistical entropy, and essentially form the "toolkit" of information theory. They include: *joint entropy*, *conditional entropy*, *mutual information* and *entropy rate*.

In the following section, we make these associations and determine that it is possible to develop a reasonable association of the entities of human cognition to the mechanisms of organodynamic theory; and therefore to create an organodynamic model of cognition. We shall then proceed to outline the development of such a model.

Toward An Organodynamic Model of Cognition Using fRMI Data

The initial task of developing an organodynamic model of any application domain is to identify entities in the target domain space that can be mapped to the entities of organodynamic theory. The mechanisms of organodynamics that must so mapped are these:

- The Underlying System of Cognitive Elements
- Cognitive Organodynamics State
- The Cognitive Organodynamics Sample Space (OSS)
- The Cognitive Organodynamic Probability Space (OPS)
- The Cognitive Organodynamic Probability Distribution (OPD)
- The Cognitive Organodynamic Dependent Probability Distribution (ODPD)

- The Cognitive System Process
- The Cognitive Organodynamic Stochastic Process (OSP)
- The Cognitive Dependent Organodynamic Stochastic Process (ODSP)

The mathematical equipment that we provided regarding "system dynamics" in the previous article, and within the MCD example, needs some enhancement. The ability of our theory to make statements about the future time evolution of a dynamical system that it is modeling is somewhat wanting.

Given that organodynamic is nondeterministic by design, we do not require that its dynamics predict exactly such a time evolution. However, it should have the necessary equipment to be able to mathematically constrain and predict that behavior within qualified parameters. We shall refer to these mechanisms as "stochastic regulation and prediction dynamics." Parts IV and V of this series of articles elaborate a considerable amount of research already accomplished with organodynamics research in this area.

The Underlying System of Cognitive Elements

The most basic idea in organodynamics is that of an entity that organodynamics calls an *element*. An *element* represents the smallest unit of identity that organodynamics models. So, the first step in developing our model of cognition using fRMI data is to decide which of all of the many object types identified in the [Turk-Browne 2013] Science article we shall use as our "elements". These should be the simplest of the object types discussed in the Science article.

The smallest unit of observation by MRI brain scans – as represented in the [Turk-Browne 2013] article is *neuronal activation*. Neuronal activation refers the "firing" by a neuron of an electrical signal. This is the most basic type of observation that fRMI seems to observing and collecting, so we shall choose such an event, neuron activation, as our *element*. In fact, for simplicity, we shall abbreviate this event to the term "neuron".

It should be noticed that this choice is not the only one we could have made. There is no need to take the time to consider other choices for our "element" type right now. But it is important to understand that the decision as to which phenomenon within one's application domain is to be the *element* is a decision that has consequences. And, more often than not, there are multiple reasonable choices. If one makes the "wrong choice" for the *element*, it will be evident soon as one defines the remaining mechanisms of organodynamic in term of this choice. If this choice is "wrong" it will be revealed as dissatisfaction in these derived mechanisms. Such dissatisfaction simply means that you are not modeling what you intended to.

Once you have selected what type of entity within the domain space is to fill the role of the *element*, then the second mechanism that you must choose – the underlying set S of elements - is mostly already determined.

In our case, since the *elements* of our cognition space are *neurons* (actually, neuron activations), then our *underlying set* S is some set of neurons. What we still have to choose is the scope of that set – that is, just how many neurons are included in S? Certainly, our S should include all of the neurons in the brain that are being sampled by the fRMI data collections. The [Turk-Browne 2013] article seems to imply that it the whole brain. So we shall go with that. We shall assume that our underling set S includes all of the neurons in the brain.

Cognitive Organodynamics State

Our next steps in developing an organodynamic model of cognition, or "mind" is to develop a model of *system state*, or to define O_{TRS} and $\mathbf{O_S}$ for our model. This involves finding a super-set of our *elements* (neurons) that "makes sense" with regard to neurons, and that *covers* (or maybe even partitions) our space of all neurons.

Recall that, in the previous article, we defined our notion of *organizational state* to be a mathematical construct whose purpose is to *organize* our underlying set S in two major ways. The first way is to provide a set of "compartments" into which all of our elements in S can be apportioned. This we accomplished by initially identifying a way to partition, or cover", S into a bunch of compartments.

To accomplish the compartmentalization of S, we call upon the fact that brain science has divided the brain into "regions". Moreover, at any point in time (single fRMI "snapshot") a certain set of the neurons, each belonging to a certain set of brain regions, are active; while the other neurons in each of those brains regions are inactive. The intersection of the set of all active neurons at a single time step with the set of all brain regions forms a collection of subsets of active neurons at that time step. The intersection of the set of all inactive neurons at a single time step with the set of all brain regions forms a collection of subsets of inactive neurons at that time step.

The union of these two sets of neurons forms a covering C_S of the brain for a single time step. This covering induces a topology – as discussed in Part II. And, since the set of active versus inactive neurons is subject to change at each time step, then the topology T_S that describes each time step can change.

The purpose of forming this topology T_S is that it provides a way of "compartmentalizing" – compartments that are subject to change with each time step. The new embellished set of "compartments" that make up the topology are called "open sets". In the previous article, we symbolized this new set, the topology, with T_S.

This topology, T_S, completes our mechanism for our first way of organizing the elements (neurons) of S. It does this by providing a very complete set of "compartments" into which to apportion the neurons in S.

But, "apportionment" is only one of two ways that we defined in Part II to "organize" S, our underlying set of neurons. But in organodynamics, we have a second way – a way that depicts any interrelationships among the neurons. The second way that we "organize" our underlying set of neurons is to take all of the neurons within each of the open sets, and create a *relation*, on them. This relation defines how those neurons are interrelated within its open set.

So, at this point, we have "apportioned" all of our neurons across all of the open sets in our topology T_S. Moreover, for each such open set, we also have a *relation* on that open set that depicts how the neurons in that open set are interrelated. Thus, we have two "things" define for each open set in our topology: 1) the open set itself, consisting of some neurons, and 2) a *relation* on that open set that depicts how its neurons are interrelated with each other.

But how are we, in this model, to define when neuron A is related to neuron B within an open set of neurons? We shall make this assignment by referring to the research described in the [Turk-Browne 2013] article. The article points out that contemporary fRMI-based cognitive research is capable of detecting *interactions among neurons*, when these "firings" occur, rather than merely that a neuron has fired. But each of

these neuronal interactions specifies a set of pairs of neurons. This is because, when a neuron fires, it sends signals to (usually) multiples of other neurons (and maybe even to itself). One pair for each recipient neuron can represent this event. Each of these pairs has as its second entry the recipient neuron; and as its first entry the firing neuron.

Of course, at any point in time, multiple neurons in an open set may be firing. And each of these firings has an entire set of pairs that represent that firing. Therefore, the complete picture of these neuronal interactions for that open set can be represented by the collections of all of these firings for that open set. This is how we shall define the *relation* of neurons on an open set.

What we are going to do next is: 1) For each of those open set, we are going to form a pair whose first entry is the open set itself, and whose second entry is its relations. 2) Form a new set whose members are all of these pairs. In the previous article, we symbolized this set as T_{RS}.

We then paired each open set of the topology T_S with its corresponding relation on that open set to obtain the construct O_{TRS}, which we described as an "extended topology on S". It is this construct that we use to represent an *organization of S*.

Thus, we have succeeded in defining the structure O_{TRS} that shall represent the (instantaneous) *organization of the brain* that we shall use in our organodynamic model of cognition. It is this structure that represents "instantaneous cognition". That is, the set S of neurons and its organization into *brain regions* (our *cover* of S) represents "the brain"; while the structure O_{TRS} represents (instantaneous) cognition – which is our definition of *system state* in this model.

Cognitive Composite Organodynamics State

In the previous article, we described how we are to represent multiple "levels of nesting" within our O_{TRS} structure. We did this by identifying subspace topologies within the topology T_S of the construct O_{TRS} in the model. This construct can also identify subspaces of neurons in the brain that behave as cognitive subspaces. Further research in the area of cognition is need to determine the applicability of this construct to human cognition.

The Cognitive System Process

Of course, we are interested in modeling cognition. In these papers, we shall regard cognition, or "mental function", as a process in which the organization of neuronal activation changes over time. We believe this is a reasonable interpretation of cognition as discussed in the [Turk-Browne 2013] article.

Under this assumption, our organodynamic model shall represent cognition as a *sequence of instantiated O_{TRS} structures*, one for each time step in the time interval of interest. By "instantiated", we mean that a structure of the same form, O_{TRS}, constitutes a time step of the process. However, the "content" (parameters representing the pattern of neuronal activation) of that form is subject to change at each time step. This fact reflects that the subject's "pattern of mental function" is different at each time step. For example, "change of thought" is what we call "thinking". Sometimes, without loss of generality, we shall refer to cognition as "thinking", even though it also includes other mental activity such as remembering, understanding and learning.

Of course, such a description depicts a *deterministic system process* in which the precise O_{TRS} structure is already realized. Such a process could reasonably model cognition "after the fact", when the thoughts have already manifest. This is a view of past cognitive events. Or it is a view of cognitive events in which though over time is pre-determined.

However, we know that a specific thought does not always lead to the same "next thought". And in our model, the occurrence of the same sequence of O_{TRS} structures does not always lead to the same O_{TRS} instance. In other words, our model of cognition must be nondeterministic. That is, we regard cognition as a nondeterministic process.

Moreover, in organodynamics, the approach that we take to modeling nondeterminism is *stochasticism*. Thus, thus we must introduce the necessary mathematics to represent stochasticism into our model. This will be accomplished through structures from probability theory, information theory and stochastic processes.

Over the next three sections, we shall gradually build up the necessary equipment to "convert" the deterministic *cognitive system process* that we have described in this section to a stochastic version, that we shall call a *cognitive organodynamics stochastic process*, or cognitive OSP.

The Cognitive Organodynamic State Space

In organodynamics, *state* is defined as the *system state*. So far, we have defined systems state to be a structure of the form O_{TRS}. And the set $\mathbf{O_S}$ of all possible entities of the form O_{TRS} we have defined to be the *state space* of an organodynamic system.

Lets point out a few properties of this space – especially for our organodynamic cognition model. It is generally a very large and rich space. The underlying set S of all neurons has billions of members. Thus, the state space $\mathbf{O_S}$ has many trillions of members.

Moreover, it has a great deal of structure – for example we have already shown how to define a family of topologies on it. And we defined the notion of a *component* of this space being a topological subspace of $\mathbf{O_S}$. And, since these component subspaces can be "nested" (composed), then we have a rich topological space with many subspaces of subspaces.

The Cognitive Organodynamic Probability Space (OPS)

The *organodynamic state space* (OSS) is the final *static* mechanism in the organodynamics theory. In the case of our model of cognition, the OSS is the set $\mathbf{O_S}$ of structures of the form O_{TRS}, each of which represents exactly one logically possible way that our neurons in S can be organized.

But, the OSS is also the bridge into specifying the dynamical mechanisms of organodynamics. This is accomplished by specifying the OSS as the sample space $\mathbf{\Omega}$ part of a probability space. This effectively means that we are going to assign a probability value to each of *states*, O_{TRS}, in our state space $\mathbf{O_S}$. That is, we shall effective assign to each of O_{TRS} a probability value. As we are about to discuss, this assignment will be the beginnings of embellishing our state space $\mathbf{O_S}$ into a probability space.

Once the transition has been made, we shall start to refer to the *states* as *sample points*, and to the *state space* as the *sample space*. Of course, our sample space $\mathbf{O_S}$ is finite – very large, but finite.

A *probability space* [Ash, et. al. 2000] is a complete formal system that establishes all that is necessary in order to work with probabilities on some system of elements. Suffice it to say that a probability space is a formal system that consists of three parts:

1. $\mathbf{\Omega}$: A set of elements, x_i, treated as sample points
2. F: A sigma-algebra on those sample points
3. ρ: Probability assignments on the sigma-algebra

Here, $\mathbf{\Omega}$ is our set $\mathbf{O_S}$ of *organization entities* O_{TRS}. F is a collection of subsets of $\mathbf{O_S}$, to which probabilities will be assigned. We shall insist that all O_{TRS} are represented as members of Ω by their singleton sets. This will ensure that all O_{TRS}'s effectively have probabilities.

ρ is a set of probability assignments to each sample point in Ω. Additionally, the normalizing assumption is made that the probabilities of all disjoint collections of sample points in Ω that union to Ω sum to one.

The Cognitive Organodynamic Dependent Probability Space (ODPS)

The ODPS uses a conditional probability distribution. This means that, instead of understanding the sample space as a collection of O_{TRS} entities, we must now understand the sample space as a set of stochastically dependent pairs of O_{TRS} entities: of the form (O_{TRSi}, O_{TRSj}). Here, O_{TRSi} occurs at time t_n, while O_{TRSj} occurs at time t_{n+1}.

In other words, we have a joint probability space and a joint distribution. Specifically, this is a distribution of pairs of events, where the first member of each pair represents an occurrence of an O_{TRS} at time t_n, while the second member pair represents an occurrence of an O_{TRS} at time t_{n+1}.

The advantage that using a joint distribution instead of a one-dimensional distribution is that stochastic dependence can be accounted for. And, as we shall see later, stochastic dependence can bring stable behavior to stochastic processes – a property that can bring self-regulation.

Of course, in an organodynamic joint distribution, the sample points are *pairs* of O_{TRS} – not single O_{TRS}. Moreover, the probabilities of these pairs cannot necessarily be calculated. So there is the same problem as before in assigning the probabilities. Of course, the joint sample space is larger than the one-dimensional sample space because it has has N^2 sample points, where N is the number of sample points in the one-dimensional sample space.

Despite the fact that the probabilities of the joint space are more numerous, the ability to account for effect that stochastic dependence can have - in reducing the uncertainty involved in the process – brings worthwhile advantages.

Consequently, the ODSP will be the principle stochastic construct used in organodynamics for modeling system dynamics.

The Cognitive Organodynamic Probability Distribution (OPD)

A "probability distribution" is a mathematical construct that serves as an abbreviation of all the information in a probability space. A probability distribution generally enables an obvious way to associate probabilities with the sample points of the space.

For continuous sample spaces, a probability distribution is not generally able to associate a specific probability value with a specific sample point. However, it is often able to specify the sum of probabilities of all of the sample points less than a specified sample point. The implication here is that some random variable associates real numbers with sample points – thus enabling this "less than" relationship to exists.

The Categorical Distribution

However, the situation is more precise for probability spaces with finite sample spaces, because it is generally mathematically possible to assign a precise probability value to each sample point in the space. Fortunately, this is (for now) true of organodynamics, because we are always dealing with a finite sample space (usually very large, but finite).

For finite sample spaces, the kind of probability distribution that is often used is referred to as a *categorical distribution*. Essentially, a categorical distribution is an abbreviation of a finite sample space that associates a probability value with each sample point.

With finite probability spaces and categorical distributions, there is no need for a numerical value assignment to the sample points. (That is, there is no need for a random variable.) A widely used example is that of the coin toss. Here the sample points are "heads" and "tails". For a fair coin, where all outcomes (sample points) are equally likely, the probabilities are ½ for both heads and tails. Notice that there is no need to assign any numerical values to the sample points here. We can assign the probabilities directly to the sample points.

The same is true for our model of cognition. Here the sample points are the *organizational entities* O_{TRS}. Each of these logically possible O_{TRS} will have a probability value assigned to it. The result of this assignment of probabilities is the *categorical distribution*, which pairs probabilities with each of the sample points.

However, there is no need to assign any other numerical value to any of these O_{TRS} structures. That is, there is no random variable for this space.

Assignment of the Probabilities

From a practical perspective, one of the biggest tasks involved with developing the probability distribution of our cognitive model is the determination of the probability assignments to the O_{TRS} sample points. Since the sample space $\mathbf{O_S}$ literally has billions of sample points, this is clearly an intractable task.

Organodynamics is not the first probabilistic theory to be faced with this overwhelming problem. For example, statistical mechanics and quantum mechanics deal with the same issue. How do they approach this problem? Theoretically. This means that they come up with good theoretical reasons for deducing what the probability distribution should be. Statistical mechanics has come up with a number of theoretical distributions to use in its experiments, based upon certain conditions. The experimenter must

determine which of these experiments prevails for their research, and then choose to employ the specified distribution.

If repeated experimentation reveals that the theoretical assumption of these probability distributions results in low fidelity, then the initially chosen theoretical distribution is "tweaked" until satisfactory results are achieved.

Admittedly, the assignment of the proper probabilities to the sample space is problematic. And it deserves more thorough research than has been given it so far for organodynamics. However, this problem is not peculiar to organodynamics.

Entropy of the ODP

In a deterministic process, there is no evident uncertainty because the dynamics of the system precisely determine the outcomes at all future time steps. But in stochastic processes, there is uncertainty about future outcomes. Just "how much" uncertainty is the question we raise now.

Depending upon the specific nature of a specific stochastic process, there may be varying degrees of uncertainty. We need to be able to quantify the degree of uncertainty involved in order to be able to ascertain whether a particular process is going to be wildly chaotic, completely unpredictable and completely untrustworthy. If so, then any model of it may be of limited utility.

On the other hand, if a particular stochastic process can be shown to be "reasonably well behaved", then a stochastic model of it can be of considerable value.

Thus, what we need is some function that measures the degree of uncertainty inherent in a probability distribution. Such a function should map a probability distribution to a real number, where that real number represents a measure of uncertainty of the probability distribution. In other words, such a functional has as its domain space a set of probability distributions, and as its codomain some set of real numbers.

An offshoot of probability theory - *information theory* - has defined such a function. It is called *statistical entropy*. We shall often refer to it as *stochastic entropy*, or simply *entropy*[1].

As a precursor to defining *entropy*, information theory first defines a simpler notion: *uncertainty*, u(x), which maps a single sample point x of a finite distribution to a real number. This functional u(x) is a measure of the degree of uncertainty of a single sample point. It is calculated by $u(x) = 1/\log(p(x))$, where p(x) is the probability of x according to a specified distribution, and the logarithmic base is specified.

Clearly, u(x) varies indirectly as probability. That is, as the probability of a sample point decreases, its degree of uncertainty increases (and vice versa). This fits with our intuition regarding the relationship between "likelihood" and "uncertainty".

We can now define the entropy H(X), where X is a categorical distribution. It is simply the mean of all u(x) over the sample space. Thus, the entropy of chance variable X is H(X), which represents the degree of uncertainty inherent in probability distribution X:

[1] Statistical entropy is not to be confused with thermodynamic entropy. The former is an analog of the latter, but it is not equivalent. Especially, it should be noted that statistical entropy is stochastic; whereas thermodynamics entropy is deterministic.

$$H(X) = \sum_{i=1}^{n} p(x_i) log(\frac{1}{p(x_i)})$$

$\forall\ x_i \in \mathbf{\Omega}$, where $\mathbf{\Omega}$ is a finite sample space.

This expression is usually simplified to

$$H(X) = -\sum_{i=1}^{n} p(x_i) log(p(x_i))$$

The Cognitive Organodynamic Stochastic Process (OSP)

Recall from above that we defined an *organodynamic system process* to be a sequence of *system organization* O_{TRS} instances, where each step in the sequence represents a moment in time. (For now, we are confining our investigation to a discrete-time process, or time series.)

Review of the Cognition State Space

Before we proceed with defining the organodynamics stochastic process for organodynamic model of cognition, it may be helpful to review how we have thus far modeled the static aspects of cognition using the more static mechanisms of organodynamic machinery.

Each of these O_{TRS} describes a logically possible instantaneous organization of the whole brain. This means that each O_{TRS} is a snapshot of neuronal activity and neuronal interaction at one moment in time. Recall that any O_{TRS} in our cognition model is a complex structure that is essentially an extension to a topology on the space of neurons. It represents the way the mind organizes the brain at an instant in time to pockets of activity across the entire set of neurons of the brain.

An O_{TRS} is developed by starting with a topology T_S on S that provides the compartmentalization of the neurons in the brain into pockets of activity and inactivity within each of the regions of the brain. But we also want to know which neurons within and across these pockets are intercommunicating. This is accomplished in organodynamics by describing these network relationships via a set-theoretic *relation*. For each open set of neurons in T_S, such a relation describes the network of interrelationships among all of the neurons in that open set. Then, each of these relations is paired with its associated open set in the topology. Thus, O_{TRS} becomes a set of pairs (X, Y) as follows:

O_{TRS} = { (X, Y) | X is an open set in T_S, and Y is a relation on X. }.

Having reviewed the nature of an O_{TRS}, we can now proceed to discuss ODSPs in the cognition example.

The Organodynamic Stochastic Process for Cognition (OSP)

We shall define an OSP by starting with an organodynamic systems process – which we defined earlier – and then alter it so that it becomes an OSP.

Recall that an organodynamic systems process is a sequence of O_{TRS} structures, one for each realized time step in the process. And, this deterministic model predicts exactly one of a possible O_{TRS} for each specific time step. (Or, the entire system process may represent which O_{TRS}'s were realized after-the-fact.)

However, our stochastic model, for each time step in the process, needs to represent the set of all possible O_{TRS}'s that can possibly be manifested at that time step – along with the relative weights (probabilities) for each of those O_{TRS}'s for that time step.

This can be achieved by replacing the exact O_{TRS}'s instances at each time step in the system model with an organodynamics probability distribution (OPD) for that time step. In other words, our OSP for modeling cognition is a sequence of OPDs, one for each time step in the cognitive process.

Since an OPD is essentially an abbreviated version of an organodynamic probability space (OPS), then more formally we could define an *organodynamics stochastic process as a sequence of OPSs*, one for each time step in an organodynamics process.

Piecewise Homogeneous OSP

Very often, as long as the underlying set S of all neurons in the brain remains unchanged, then the same OPD (and OSS) faithfully represents all time steps of an OSP. Whenever this occurs it is mathematically fortunate, because we shall be able to bring to bear a number of theorems from the theory of finite stochastic processes to assert a body of statements about the time evolution of the stochastic process, and the phenomenon being modeled.

However, more often, the underlying set S will remain the same for some finite time period, then change and remain the same for some other finite time period, and then continue to repeat this kind of behavior. This results in the same OPD appearing in some contiguous set of time steps within the OSP, followed by a change in OPD (to reflect the new set of neurons in the brain), followed by the same new OPD appearing in some contiguous set of other time steps, etc. We call this kind of behavior a *piecewise homogeneous stochastic process*.

Of course, this kind of behavior is very general, and also applies to a couple of special (degenerate) cases. The first is the case when the same OPD works for all steps in the process; and the second special case being when the OPD changes at every step of the process. Thus the idea of a *piecewise homogenous OSP* actually covers all of the cases we are interest in. Consequently, without loss of generality, we may as well define the concept of an OSP as a piecewise homogeneous stochastic process.

> Definition: Organodynamics Stochastic Process (OSP): a piecewise homogeneous stochastic process whose probability distributions at each time step are *organodynamic probability distribution* (ODPs).

The condition that the probability distributions be ODPs simply means that their sample spaces are organodynamics state spaces – are some $\mathbf{O_S}$.

In Part IV we shall exploit these kinds of conditions and further employ the entropic functionals of information theory to build a stronger system dynamic into organodynamics as a dynamical systems theory, and to introduce the concept of prediction and system regulation into this theory. This will be very important, because the general applicability and significance of any dynamical systems theory is to provide

a mechanism of dynamics that enables predictability of the time evolution of the systems that it models.

One difference between organodynamics and other dynamical systems theories is the ability of organodynamics to provide a prediction mechanism even in the face of various degrees of uncertainty. Of course, along with this, organodynamics must provide a measure of the degree of fidelity one can expect from any particular organodynamic model, based on the degree of uncertainty inherent in the system being modeled.

Measuring Behavioral Dynamics

Because of their stochastic nature, OSPs may exhibit wildly unpredictable behavior – or they may settle down to an acceptable range of chance variation. For organodynamics to be useful for application modeling, it must provide some dynamics that 1) ascertain the degree of predictability of particular applications, and 2) describe a mechanism of dynamics that provides for reasonably accurate predictions in cases where there is a degree of predictability.

In other words, just because of stochastic reasons alone, it is often the case that something useful can be said about the stochastic time evolution of the system into the future. It is the responsibility of organodynamics, in its capacity of a stochastic dynamical systems theory, to describe the condition under which prediction is realistic, and when so, how to make those predictions, and the error bounds around making them.

We shall take up these issues in the next article of this series, Part IV. We shall be looking for aspects of probability spaces that impose various types of constraints on system behavior over time. These constraints have the effect of reducing degree of uncertainty of various aspects of future behavior of stochastic dynamical systems, and therefore increase their predictability. This kind of reduction of uncertainty is always revealed by stochastic *entropy* or by one of its several *entropic functionals*.

Therefore, information theory – the study of these entropic functionals - will become a mathematical foundation of the dynamics of organodynamics. So for now we shall simply say that at that time we shall leverage the entropic functionals of information theory to provide the mathematical foundations of these dynamical aspects of organodynamics.

The Organodynamic Subprocess for Cognition

So far, we have constrained our organodynamics model of cognition to necessarily model the whole brain – or at least to include all of the neurons that lie within the scope of an fMRI experiment.

What we would like to do now is to show how organodynamics can provide models of mental subprocesses within the overall process of cognition being measured by such an experiment.

In other words, we expect that cognition should be able to be “broken down” to reveal many concurrent “strains” or “threads” that operate in parallel. Our organodynamic model should be able to focus on any one of these. Note that the model we have developed so far already includes them all (simultaneously). What we also want is the ability to focus on any one of them

Moreover, any of these "threads of mental activity" may be of any level of scope. This is because the brain has composite organization. There are systems within systems within systems in the brain. Therefore, there are levels within levels within levels of cognitive process and cognitive activity occurring concurrently within the "mind".

Consequently, whenever we "focus" on any one of these – as we are discussing here, we want also to be able to choose which level of scope – which level of organization – we want to focus on. Once we choose a level of cognitive activity to model, we further expect any other cognitive threads that are subsumed, as sub-processes, within the chosen level of cognition will also be included in the model.

It turns out that all of this capability occurs naturally simply by considering the topological nature of the way we have defined *system state*, O_{TRS}, in this theory. The reader will recall that we established the machinery that enabled the idea of *nested systems* with our notion of systems state O_{TRS}. We did this by defining the idea of *component* of a topological space being a *topological subspace*, or *subtopology*, of the topology T_S in the model.

We further showed how a subtopology of T_S can further have subtopolgies nested within it. And when we do, the sub-subtopology is also a subtopology of T_S. That is, this definition of *components* as subtopolgies establishes a partial order relation on these components. They can be "nested" through these subtopology relationships.

So this idea of *components* as subtopolgies provides the mechanism that we need for modeling sub-process of cognition.

In fact, since the whole brain is a subprocess of itself, then using organodynamics to model cognitive subprocess is precisely the same exercise as modeling the cognition involved in the whole brain.

Moreover, an organodynamics model of the whole brain is *already* a joint model of all of its cognitive subprocesses. There is no need to invent any new machinery to "combine" any such cognitive submodels into an integrated whole cognitive model. That integration already exists in the form of the overall organodynamic cognitive model involving the whole brain. There is no other form that such an integrated model should take over the form of the model of the whole system.

The Organodynamic Web

In the previous section, we described a kind of system reorganization that went further than mere reorganization of components. In addition, this type of reorganizing also partitioned the reorganized components into distinct subprocesses. Where there was one process that exhibited reorganization over time, there were now two (or more) concurrent (or simultaneous) processes, each of which operates some piece of the newly partitioned reorganization of components.

This new aspect of reorganization introduces the notion of *splitting* the newly reorganized components, at some time step in the organodynamic process, into multiple distinct processes. Mathematically, this kind of action represents a type of *unary algebraic operation* that maps a single process (at one time step) to a tuple of processes at the next time step. Specifically, it maps a single organodynamic stochastic process (OSP) to a tuple of OSPs at a time step.

Conversely, one can imagine the notion of two concurrent OSPs coming together at one time step to produce a single unified OSP at the next time step. If the previous

paragraph describes a kind of "forking" OSP, then the present paragraph can be understood to describe a kind of "joining" OSP. These "joining" OSPs can be defined mathematically as a kind of binary (or n-ary) operation on OSPs that produces an "output" OSP – all of this occurring at a specific time step.

Of course, if we combine these ideas, we can develop the idea of a network of OSPs – which we shall call an *organodynamic web*.

These ideas have been explored elsewhere [HollandJG-II 2011], and we shall not develop them here – mainly because of limitations of space and time. Hopefully, the intuition of a network of organodynamic processes, involving the "forking" and "joining" of OSPs at various "nodes" (time steps), is strong enough at this point that we can refer to it later. And, we shall refer to these ideas in Part V.

The Cognitive Organodynamic Dependent Stochastic Process (ODSP)

Recall that an *organodynamic probability distribution* (OPD) is a categorical probability distribution whose sample space is $\mathbf{O_S}$.

In order to account for stochastic dependence in organodynamics, we would like a version of the OSP wherein the OPDs within the OSPs are replaced by a version of the OPD that is a joint distribution. We shall call such a joint OPD an *organodynamic dependent probability* distribution, or ODPD.

In such a case, the resulting OSP will have time steps that are ODPDs rather than OPDs. We shall call such a process an *organodynamic dependent stochastic* process, or ODSP.

The ODSP will be the principle mathematical construct that will be used to model complex adaptive dynamical systems in organodynamics. Essentially, all previous mathematical machinery that we have developed above exists for the purpose of being able to define the concept of the ODSP.

It further turns out that any such stochastic dependence between a specified time step and the outcomes of previous time steps are revealed by the *entropic functionals* of information theory. But more than that, the *degree of this stochastic dependence* is also *measured* by these entropic functionals.

As I discussed in Part II, we typically represent an ODSP as a sequence of Markov transition matrices, one for each time step of the process. We have discussed that an organodynamic process is typically *piecewise homogeneous*. This means that the same probability distribution (ODPD) holds for some finite consecutive subsequence of time steps. This fact means that an ODSP can be represented as a sequence of Markov transition matrices, one matrix for each change in probability distribution at. Thus, each of these transition matrices holds for all of the consecutive time steps that it models.

Thus, the entropic functionals of information theory that we have discussed above, then, apply to each of these Markov matrices in an ODSP. Thus, the behavior of these entropic functionals for a given stochastic process, or ODSP for our theory, can portend the predictability or non-predictability of the process. And not only can these entropic functional assess that predictability, but the can also measure it degree as well.

Conclusions of Part III

In this article we have used the data provided by the [Turk-Browne 2013] article in November 2013 Science magazine to provide an fRMI-based model of cognition of the human mind. What we have hopefully demonstrated is that organodynamics provides a system of modeling mechanism from which to construct a stochastic model of the way the human cognition can be understood as the reorganization of neuronal activity and interaction over time.

In this article, we revisited a number of mechanisms of organodynamics that were first presented in the previous article, Part II of this series. And we also presented a number of new mechanisms that were not mentioned before.

In particular, we elaborated on the application of ideas from topology to define how system organization in organodynamics can be defined as an elaborated state structure. In particular, topology enables us to represent the manner in which the elements of an underlying space can be "apportioned" across a set of "compartments". However, a second meaning of "organization" involves describing the interrelationships among elements. To include this meaning, we "decorated" each open set of the topology with a relation on its elements.

These mechanisms provided a model of the static aspects of systems. We the further developed the dynamical mechanisms of organodynamics. These include various ideas that are necessary to be able to define the concept of the stochastic process, and finally of the dependent stochastic process. For the purpose of organodynamics, however, such probabilistic mechanism must always have as their sample space the set of all possible *organizations* of the underlying set of basic entities. This fact is what makes a probability distribution be an *organodynamics probability distribution* (OPD), a stochastic process be an *organodynamics stochastic process* (OSP), and a conditional stochastic process be an *organodynamics dependent stochastic process* (ODSP).

Finally, we suggested that we need a way to assess whether or not an ODSP is predictable, or not; and if so, then how much so. This is an important consideration, because without some mathematical equipment to make these assessments and characterizations of a dynamical system being modeled, it would be stretching credulity to call organodynamics a true *dynamical system*. Certainly any dynamical system worth of the appellation must provide mechanism for some kind of prediction.

Heretofore, we have suggested a number of stochastic mechanism by which we would eventually provide a true dynamics, but the actual dynamics has not really been forthcoming. And they still have not at this point. What we have done here, however, is to suggest that we are going to use the mathematics of information theory, *entropic functionals*, to provide this foundation – in Part IV, entitled "Prediction Dynamics and Stochastic Regulation".

Preview of Part IV

In Part IV, we shall look at how information theory provides a framework that organodynamics can leverage in order to ascertain the conditions under which an organodynamic process can exhibit limiting, predictable long-run behavior.

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